

# Sequence Lec-01

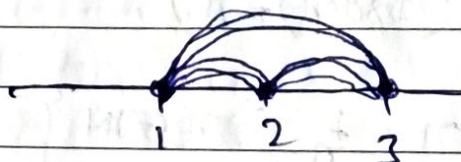
L-20

Defn: A sequence is a function  $f: \mathbb{N} \rightarrow A$  s.t.  
 $f(n) = (f_n) \in A.$

$\rightarrow$  A sequence  $f: \mathbb{N} \rightarrow \mathbb{R}$  is called a real sequence defined by  
 $f(n) = (f_n) \in \mathbb{R}.$

\* Range of sequence:

$\langle 1, 2, 3, 1, 2, 3, 1, 2, 3, \dots \rangle$



Range of sequence = Ball  $\cup$  Ball  $\cup$  Ball  $\cup$  ...  
 $= \{1, 2, 3\}.$

(\*) Bounded sequence:

$\langle (-1)^n \rangle = \langle -2, 2, -2, 2, -2, 2, \dots \rangle$



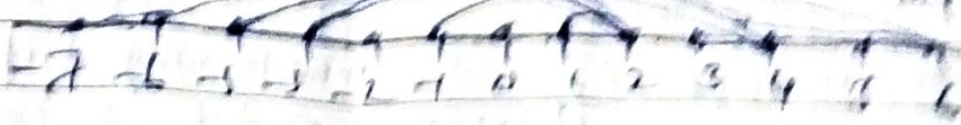
हरेक बिन्दु के bound point  $-1$  &  $3$  के बीच है  
इसलिए this sequence is bounded.  
हरेक sequence bounded नहीं हो सकती है।  
ex:  $\langle 1, 2, 3, 4, 5, 6, 7, \dots \rangle$  is unbounded.

but lower bound भी हो सकती है (like  $-1, 0, 1, \dots$  etc)

$\Rightarrow$  if is bounded below sequence.

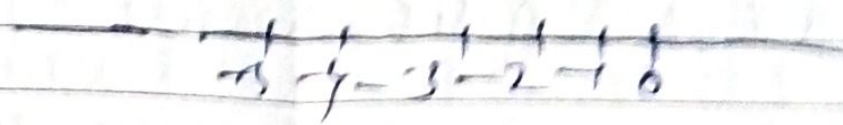
$$\langle \mathbb{N} \rangle = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

is bounded (non-bdd) below



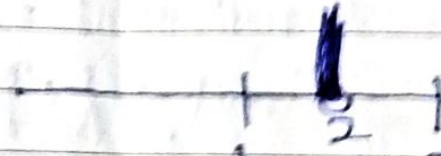
$$\langle -\mathbb{N} \rangle = \{-1, -2, -3, -4, \dots\}$$

is bdd above but not bdd below



### Constant sequence

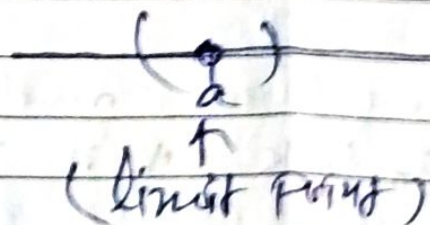
$$\langle 2, 2, 2, \dots \rangle$$



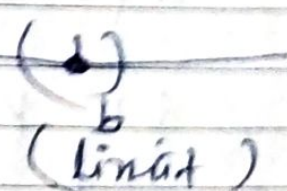
is a constant sequence is bdd.

→ every constant sequence is bdd but every bdd sequence is not constant sequence.

→ limit & limit point  
(unique) (may not be unique)



every nbd of 'a' contains infinitely many points of



every nbd of 'b' contains infinitely many points other than 'b'.

→ every limit is a limit point

converse need not true.