P.G. Semester - II, CC - VI

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Complex Analysis

Ques - State and prove Abel's theorem of power series.

Ans -

Statement - If the power series

$\sum_{n=0}^{\infty} a_n z^n$ converges for a particular value of z_0, then it converges absolutely for all values of z for which $|z| < |z_0|$

Proof - Let $\sum a_n z^n$ converges.

Then its n the term $a_n z_0^n$ must tends to 0 as $n \to \infty$.

So we can find a number $M > 0$ such that

$$|a_n z_0| \leq M \quad \text{for all } n$$

Then $$|a_n z^n| \leq M \left|\frac{z}{z_0}\right|^n$$

Since $|z| < |z_0|$, the geometric series $\sum \left|\frac{z}{z_0}\right|$ converges. It follows by the comparison test that $\sum |a_n z^n|$ converges for all values of z for which $|z| < |z_0|$.

In other words $\sum a_n z^n$ converges absolutely for all z such that $|z| < |z_0|$.

Proved