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Resultant of two S.H.M.s at right angles

Resultant of two S.H.M.s at right angles same period but differing in amplitude and phase-
$\longrightarrow$

Let the two S.H.M.s of equal frequencies (equal period and wavelength) along the x and y axes. If their displacements at any time t be x and y, and represented by

$$x = a\sin\omega t \qquad \text{--- (1)}$$
$$y = b\sin(\omega t + \phi) \qquad \text{--- (2)}$$

from equation (1),

$$\sin\omega t = \frac{x}{a} \qquad \therefore \cos\omega t = \sqrt{1 - \frac{x^2}{a^2}}$$

from equation (2)

$$\frac{y}{b} = \sin(\omega t + \phi)$$

$$= \sin\omega t \times \cos\phi + \cos\omega t \times \sin\phi$$

substituting the value of $\sin\omega t$ and $\cos\omega t$, We have

$$y/b = \frac{x}{a}\cos\phi + \sqrt{\left(1 + \frac{x^2}{a^2}\right)}\sin\phi$$

$$\left(\frac{y}{b} - \frac{x}{a}\cos\phi\right) = \sqrt{\left(1 - \frac{x^2}{a^2}\right)}\cdot\sin\phi$$

Squaring both sides, we get,

$$\left(\frac{y}{b} - \frac{x}{a}\cos\phi\right)^2 = \left(1 - \frac{x^2}{a^2}\right)\cdot\sin^2\phi$$

or, $$\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2}\cos^2\phi + \frac{x^2}{a^2}\sin^2\phi - \sin^2\phi = 0$$

$$\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2}(\cos^2\phi + \sin^2\phi) = \sin^2\phi$$

This represents the resultant motion of the particle, which is in general an ellipse inclined to the axes of co-ordinate.

$$\boxed{y^2/b^2 - \frac{2xy}{ab}\cos\phi + \frac{x^2}{a^2} = \sin^2\phi}$$