Null sequence :-

Let $\{a_n\}$ be a sequence and if

If $\lim_{n \to \infty} a_n = 0$ then $\langle a_n \rangle$ is called

null sequence

Ex :- $\langle \frac{1}{n^2} \rangle, \langle \frac{1}{n} \rangle, \langle \frac{1}{\sqrt{n}} \rangle$ are Null sequence

Monotonic sequence :- These are four type

(1) Monotonically increasing sequence

if $a_n \leq a_{n+1} \ \forall n$.

Ex :- $\langle n \rangle = \{1, 2, 3, 4, \ldots\}$ & $\langle 2, 5, 9, 13, \ldots \rangle$ are monotonic increasing

(2) Monotonically strictly increasing sequence

if $a_n < a_{n+1} \ \forall n$.

Ex :- $\langle n \rangle$ & $\langle 2, 5, 9, 13, \ldots \rangle$ are mono. strictly increasing sequence

(3) Monotonically decreasing sequence

if $a_n \geq a_{n+1} \ \forall n$.

Ex(*) $\langle \frac{1}{n} \rangle = \{1, \frac{1}{2}, \frac{1}{3}, \ldots\}$

$\langle \frac{1}{n^2} \rangle = \{1, \frac{1}{2^2}, \frac{1}{3^2}, \ldots\}$

are monotonically decreasing seq.

(4) Monotonically strictly decreasing sequence

if $a_n > a_{n+1} \ \forall n$.

ex(*) are also mon.

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Least Upper bound of sequence $\langle a_n \rangle$:

Let $\langle a_n \rangle$ be a sequence. and let $\exists\, u$ s.t.

(1) $a \leq u \;\; \forall\; a \in \{a_n\}$

(2) If $\exists\, u'$ another bound s.t.
$a \leq u' \Rightarrow u \leq u'$.

Ex(1)

$\langle a_n \rangle = \{1, 2, 3, 1, 2, 3, 1, 2, 3, \ldots\}$

let (i) $u = 3$. and
$a \leq u = 3 \;\forall\; a \in \langle a_n \}$

(ii) $\because \exists\, u' = 5$ s.t.
$a \leq 5 = u'$
$\Rightarrow u = 3 \leq 5 = u'$
i.e, $u \leq u'$

Then $u = 3$ is called least upper bound of $\langle a_n \rangle$

Results

(1) every cgt sequence has a unique limit

(2) If a sequence has a unique limit point then it may or may not be Convergent.

Counter ex:- $\langle a_n \rangle = \langle 1, 0, 2, 0, 3, 0, 4, 0, \ldots \rangle$
limit point $= \{0\}$ is unique
but it is oscillating infinitely (not cgt) sequence.

(3) Every Convergent sequence is bounded.
~~Converse not true~~
but bdd sequence need not be Convergent

Ex. The sequence $\langle 1, 2, 1, 2, 1, 2, 1, 2, \ldots \rangle$ is bdd