

Shell Model of Nuclear Structure.

It is known to us that electrons are arranged in different shells and orbits which may be defined by four quantum numbers. Similarly it is proposed that the nucleons are arranged in the shells having discrete energy levels satisfying certain Quantum-Mechanical Conditions.

The properties of nucleus depends on the number of protons and neutrons in the nucleus.

The protons and neutrons are packed into separate shells (the nuclear shell) within the nucleus. A nuclear shell contains a definite number of neutrons and protons, or only neutrons and only protons also. As the capacity of each level is reached, a closed shell is formed.

The filled shell is more stable than the other shells.

Nuclei with 2, 8, 20, 50, 82 or 126 neutrons or protons or both are particularly more stable. These numbers are called Magic Numbers.

In this model the motion of individual of individual nucleon is considered, hence it is also called 'Single Particle Model' which is applicable to the nucleus in the ground state.

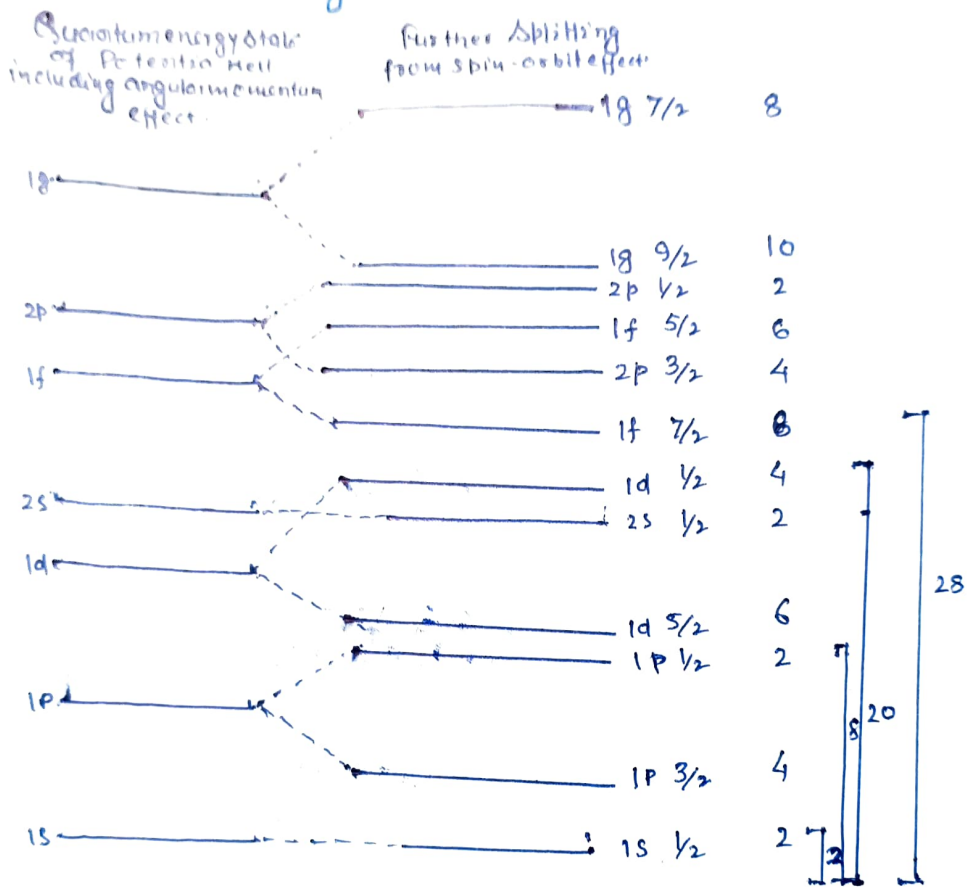
A nucleus which has filled shells is more stable than nucleus has unfilled shells. Extra stable nuclei are thus analogous to the inert gas atoms which have filled electronic shells. The elucidation of the laws governing the number of nucleons in filled nuclear shell is however, not so readily possible as in the case of electrons. Though the Pauli's Exclusion Principle applies, theoretical task is much more difficult due to following facts.

There are two different types of nucleons as compared to only one type of electrons. Unlike the extra nuclear electrons, nucleons are not subjected to the attraction of a central force of electrostatic origin. The nature of inert nucleonic forces are not fully understood.

The evidence for a kind of shell structure and a limited number of allowed energy states suggested that a nucleons move in some kind of effective potential well created by the forces of all the other nucleons. This leads to energy quantization in a manner similar to the square well and harmonic oscillator potentials.

Since the details of the well determine the energies, much effort has gone into construction of potential wells for the modelling of the observed nuclear energy levels. Solving for the energies from such potentials gives a series of energy levels.

The labels on the levels are somewhat different from the corresponding symbols for atomic energy levels. The energy levels increase with orbital angular momentum quantum number l and the s, p, d, f ... symbols are used for $l=0,1,2,3,\dots$ just like in the atomic case. But there is really no physical analog to the principal quantum number n , so the numbers associated with the level just start at $n=1$, for the lowest level associated with a given orbital quantum number, giving such symbols as 1g which could not occur in atomic labelling scheme.

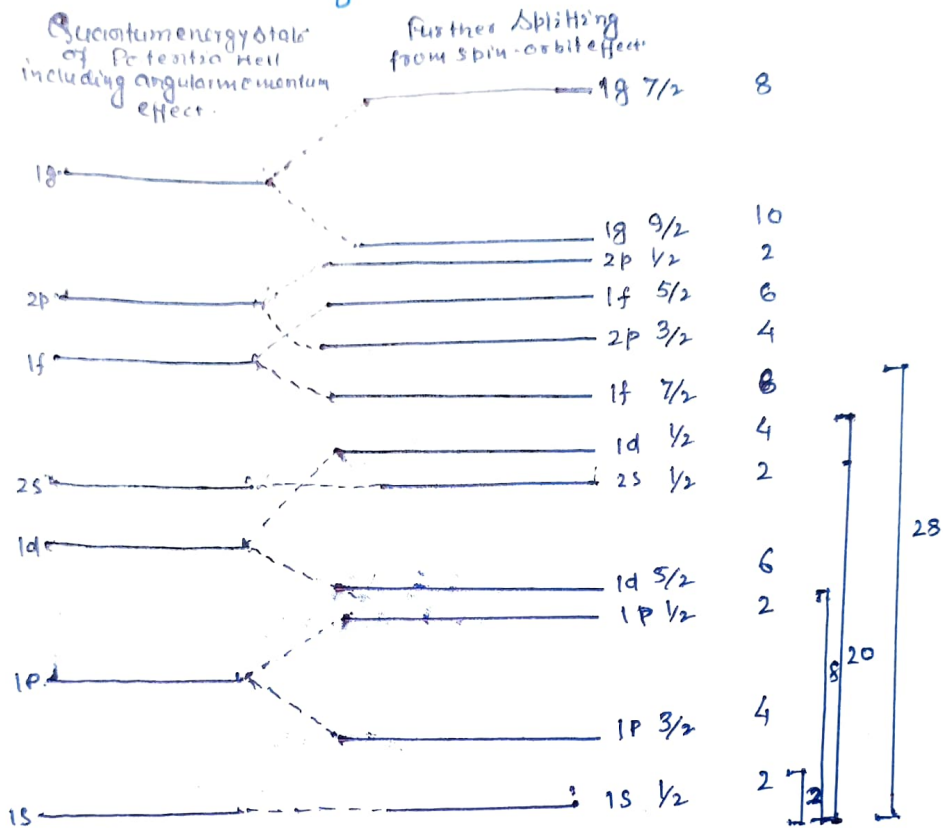


The quantum number for angular momentum is not limited to n as in the atomic case. In addition to the dependence on the details of the potential well and the orbital quantum number, there is a sizable spin-orbit interaction which splits the levels by an amount which increases with orbital quantum number. This leads to the overlapping levels.

The subscript indicates the value of the total angular momentum ' j ' and the multiplicity of the state is $2j+1$.

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With this set of identified nuclear states the magic numbers, we can predict the net nuclear spin of a nucleus. The parity of the state can be predicted, so the single particle shell model has shown it self to be significant in characterising nuclei.

Merits of the Shell Model: - The success of the shell model arises from its ability to explain the periodicity in the nuclear properties, prediction of the spin and parities of odd nuclides.

Evidence of the Shell Model: - The existence of the magic numbers i.e. 2, 8, 20, 28, 50, 82, 126 suggests close shell configuration, like the shell in atomic structure.

Other forms of evidences suggesting shell structure include: -

- (i) Enhanced abundance of those elements for which proton or neutron is a magic number i.e. ${}^8_2\text{O}$, ${}^{50}_{28}\text{Sn}$, ${}^{82}_{50}\text{Pb}$ - etc.
- (ii) The stable elements at the end of the naturally occurring radio-active series have a 'magic number' of neutrons or protons ${}^{82}_{50}\text{Pb}^{208}$.
- (iii) The mean binding energy for the elements having magic number of neutrons or protons are particularly high.
- (iv) The binding energy for the last neutron is a maximum for a magic neutron number and drops sharply for the next neutron added.
- (v) The neutron absorption cross-sections for isotopes, where neutrons are equal to magic numbers, are much lower than surrounding isotopes.
- (vi) The excitation energy from the ground nuclear state to the first excited state is greater for closed shells.
- (vii) Electric quadrupole moments are near zero for magic number nuclei.

x