

~~sequence~~  
and limit point  $\in \{1, 2\}$  (it is a limit point)

$\Rightarrow$  This sequence is not Cgt also it is not convergent

This sequence is oscillating finitely...

(4)  $\Rightarrow$  Every bdd sequence is either Cgt or oscillating finitely

(5) Every unbounded sequence never be Cgt.

ex:  $\{1, 0, 2, 0, 3, 0, 4, \dots\}$  is unbounded  
 $\Rightarrow$  sequence is not Cgt

(6) Let  $\{a_n\}$  be a sequence then and if

- (1)  $\{a_n\}$  is bdd
- (2)  $\{a_n\}$  has unique limit pt

$\Rightarrow \{a_n\}$  is Cgt.

(7) Let  $\{a_n\}$  be a sequence and if

- (1)  $\{a_n\}$  is bdd
- (2)  $\{a_n\}$  is not convergent

$\Rightarrow \{a_n\}$  has at least two limit points

(8) Bolzano Weierstrass Property: Every bounded sequence has at least one limit point.

$\rightarrow$  (i) bdd sequence is Cgt  $\Rightarrow$  one limit pt  
bdd sequence is not Cgt  $\Rightarrow$   $\forall$  it has  $\infty$  limit pt

(10) Let  $S$  be a compact set and  $\{a_n\}$  be a sequence in  $S$  then  $\exists$  a limit point of the sequence  $\langle a_n \rangle$  in  $S$ .

(11) Let  $\{a_n\}$  be bdd sequence then the collection of limit points of bdd sequence is also bdd.

(12) Let  $A' =$  set of all limit points  $\Rightarrow A'$  is closed.

(13) Set of all limit points of a bdd sequence is compact.

(14) If  $\langle a_n \rangle$  be a bounded sequence then its set of limit points may be bdd or may be unbdd.

Ex:  $\left\{ 1 + \frac{1}{2}, 2 + \frac{2}{2}, 1 + \frac{1}{3}, 2 + \frac{1}{3}, 3 + \frac{1}{3}, 1 + \frac{1}{4}, 2 + \frac{1}{4}, \dots \right\}$

is bounded sequence.

Set of limit points =  $\{1, 2, 3, \dots\}$  is closed unbounded.

L-22

Sum & product of a sequence

If  $\langle a_n \rangle$  &  $\langle b_n \rangle$  diverge to  $+\infty$  then  $\langle a_n + b_n \rangle$  is divergent to  $+\infty$ .