P.G. Sem – III, paper – CC – XIII

Classical Mechanics

Explain Jacobi's form of the principle of least action.

$$T = \frac{1}{2}\sum_{k l} a_{kl}\,\dot{q}_k \dot{q}_l = \frac{1}{2}\sum_{k,l} a_{kl}\left(\frac{dq_k\, dq_l}{dt^2}\right) \quad \text{—(1)}$$

Now assume $(d\rho)^2 = \sum_{k,l} a_{kl}\, dq_k\, dq_l$

$$\Rightarrow T = \frac{1}{2}\left(\frac{d\rho}{dt}\right)^2 \quad \Rightarrow dt = \frac{d\rho}{\sqrt{[(2T)]}} \quad \text{—(2)}$$

Hence the principle of least action implies

$$\Delta \int_{t_1}^{t_2} 2T\, dt = \Delta \int 2T \frac{d\rho}{\sqrt{(2T)}} = 0 \quad \text{—(3)}$$

$$\Rightarrow \Delta \int \sqrt{2T} = 0$$

$$\Rightarrow \Delta \int \sqrt{[2(E-V)]}\, d\rho = 0$$

$$\Rightarrow \Delta \int \sqrt{[H-V]}\, d\rho = 0 \quad \text{—(4)}$$

(Jacobi's form of the principle of least action).