Topological spaces

Def^n:- Let X be a non-empty set.
A class $\mathcal{T}$ of subsets of X is said to be Topology on X iff it satisfies the following axioms.

1. $\phi \in \mathcal{T}$
2. $X \in \mathcal{T}$
3. $\{G_\lambda\}_{\lambda \in \Lambda} \subseteq \mathcal{T} \Rightarrow \bigcup_{\lambda \in \Lambda} G_\lambda \in \mathcal{T}$.
4. $\{G_i\}_{i=1,2,\dots,n} \subseteq \mathcal{T} \Rightarrow \bigcap_{i=1}^{n} G_i \in \mathcal{T}$.

In this case $(X, \mathcal{T})$ is called a Topo. spa

EX ~~X~~ → If X is any set, (2) the collection of all subsets of X is a Topo. on X. It is called the discrete Topology on X.
(big)

→ The collection consisting of X and ϕ only is also a Topology on X. We shall call it indiscrete Topo. or the trivial Topology.

(3) EX:- Let X be a set, $\mathcal{T}_f$ be the collection of all subsets U of X such that $X \setminus U$ is either finite or all of X, then $\mathcal{T}_f$ is a Topo. on X called the finite complement Topo.

ie $\mathcal{T}_f = \{ U \subset X \mid X \setminus U \text{ is either finite or all of } X \}$

now we show $\mathcal{T}_f$ is Topology.

(1) let $\phi \in \mathcal{T}_f \Rightarrow X \setminus \phi = X$ is ~~finite~~ all of X

& $X \in \mathcal{T}_f \Rightarrow X \setminus X = \phi$ is finite

So, Both X and ϕ are in $\mathcal{T}_f$

(2) let $\{U_\alpha\}_{\alpha \in \Lambda}$ is an index family of non-empty element of $\mathcal{T}_f$. ie $\{U_\alpha\}_{\alpha \in \Lambda} \subseteq \mathcal{T}_f$

T. show :- $\bigcup_{\alpha \in \Lambda} U_\alpha \in \mathcal{T}_f$

let $\bigcup_{\alpha \in \Lambda} U_\alpha \in \mathcal{T}_f \Rightarrow X \setminus \bigcup_{\alpha \in \Lambda} U_\alpha$

$= \bigcap_{\alpha \in \Lambda} (X \setminus U_\alpha) \leftarrow$ finite

The latter set is finite, because each set $(X \setminus U_\alpha)$ is finite.

(3) let $\{U_i\}_{i=1}^{n}$ be non-empty element of $\mathcal{T}_f$. To show :- $\bigcap_{i=1}^{n} U_i$ is in $\mathcal{T}_f$

let $\bigcap_{i=1}^{n} U_i \in \mathcal{T}_f \Rightarrow X \setminus \bigcap_{i=1}^{n} U_i$

$= \underbrace{\bigcup_{i=1}^{n} (X \setminus U_i)}_{\text{finite}}$

The latter set is finite, because of the finite union of finite set is finite