Date 19/02/2025
Sem-2 Unit-2

Resultant of two SHMs at right angles with frequency ratio 1:2

Let us two S.H.Ms perpendicular to each other having differenting amplitudes and phase and frequencies in the ratio 1:2.

Let the first S.H.M be along x-axis and the second along the y-axis since their frequencies are in the ratio 1:2, their angular velocities ($\omega = 2\pi n$) will be in the ratio 1:2.

The displacements due to the two S.H.Ms at any time t ~~can be~~ can be represented as

$$x = a \sin \omega t \quad \text{---(1)}$$

$$y = b \sin(2\omega t + \phi) \quad \text{---(2)}$$

from eqn (1)

$$\sin \omega t = \frac{x}{a} \text{ and } \cos \omega t = \sqrt{\left(1 - \frac{x^2}{a^2}\right)}$$

from eq.(2) $\frac{y}{b} = \sin(2\omega t + \phi)$

$$= \sin 2\omega t \cos\phi + \cos 2\omega t \cdot \sin\phi$$

$$= 2 \sin\omega t \cdot \cos\omega t \cdot \cos\phi + (1 - 2\sin^2\omega t) \sin\phi$$

Substituting the values of $\sin\omega t$ and $\cos\omega t$, we have,

$$y/b = 2\frac{x}{a}\sqrt{\left(1 - \frac{x^2}{a^2}\right)} \cdot \cos\phi + \sin\phi - 2\frac{x^2}{a^2}\sin\phi$$

$$\left(\frac{y}{b} - \sin\phi\right) + \frac{2x^2}{a^2}\sin\phi = \frac{2x}{a}\sqrt{\left(1 - \frac{x^2}{a^2}\right)}\cdot\cos\phi$$

Squaring both sides, we get.

$$\left(\frac{y}{b} - \sin\phi\right)^2 + \frac{4x^2}{a^4}\sin^2\phi + \frac{4x^2}{a^2}\sin\phi\left(\frac{y}{b} - \sin\phi\right)$$

$$= \frac{4x^2}{a^2}\left(1 - \frac{x^2}{a^2}\right)\cos^2\phi$$

$$= \frac{4x^2}{a^2}\cos^2\phi - \frac{4x^4}{a^4}\cdot\cos^2\phi$$

$$\text{or, } \left(\frac{y}{b} - \sin\phi\right)^2 + \frac{4x^2y}{a^2b}\sin\phi - \frac{4x^2}{a^2}\sin^2\phi + \frac{4x^2}{a^4}\sin^2\phi$$

$$= \frac{4x^2}{a^2}\cos^2\phi - \frac{4x^2}{a^4}\cos^2\phi$$

$$\text{or, } \left(\frac{y}{b} - \sin\phi\right)^2 + \frac{4x^2y}{a^2b}\sin\phi - \frac{4x^2}{a^2}\left(\sin^2\phi + \cos^2\phi\right)$$

$$+ \frac{4x^4}{a^4}\left(\sin^2\phi + \cos^2\phi\right) = 0$$

$$\text{or, } \left(\frac{y}{b} - \sin\phi\right)^2 + \frac{4x^2y}{a^2b}\sin\phi - \frac{4x^2}{a^2} + \frac{4x^2}{a^4} = 0$$

This eq. gives the resultant motion of the particale. (3)

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