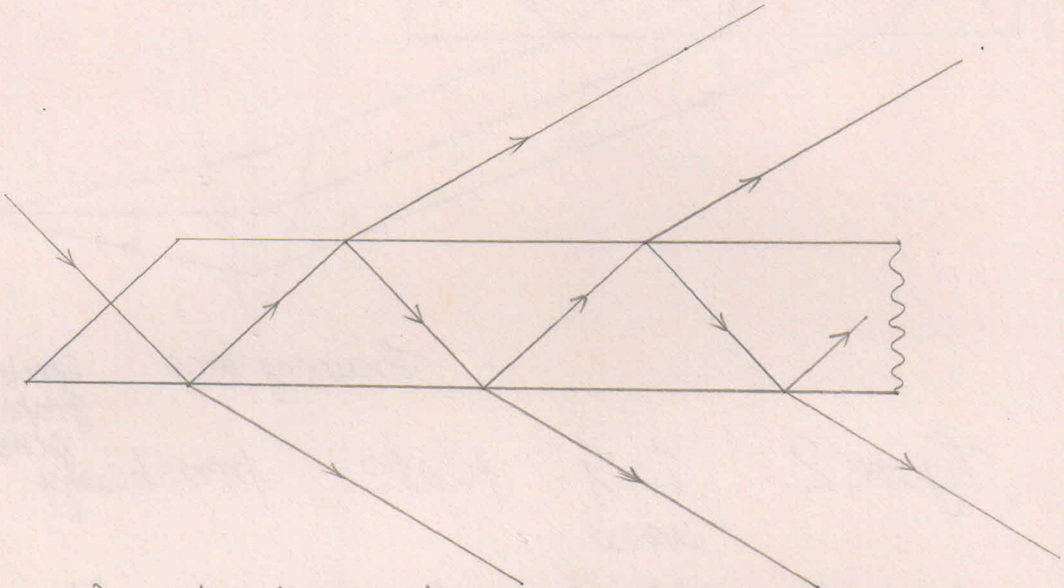
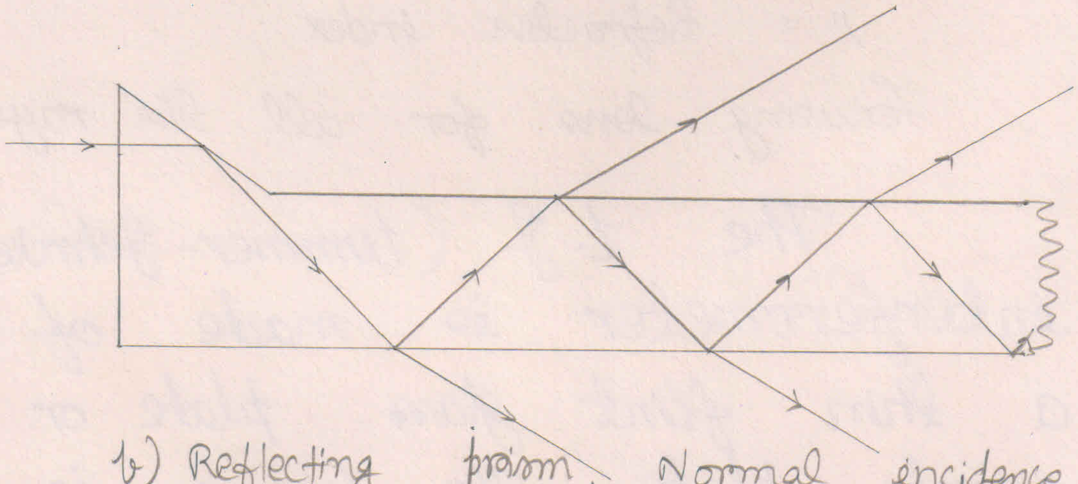
* Explain the theory of L-G plate (Lummer Gehrcke interferometer). Obtain expression for its resolving power.



a) Slanting face, Normal incidence



b) Reflecting prism, Normal incidence

Fig 1: Two types of L-G plates.

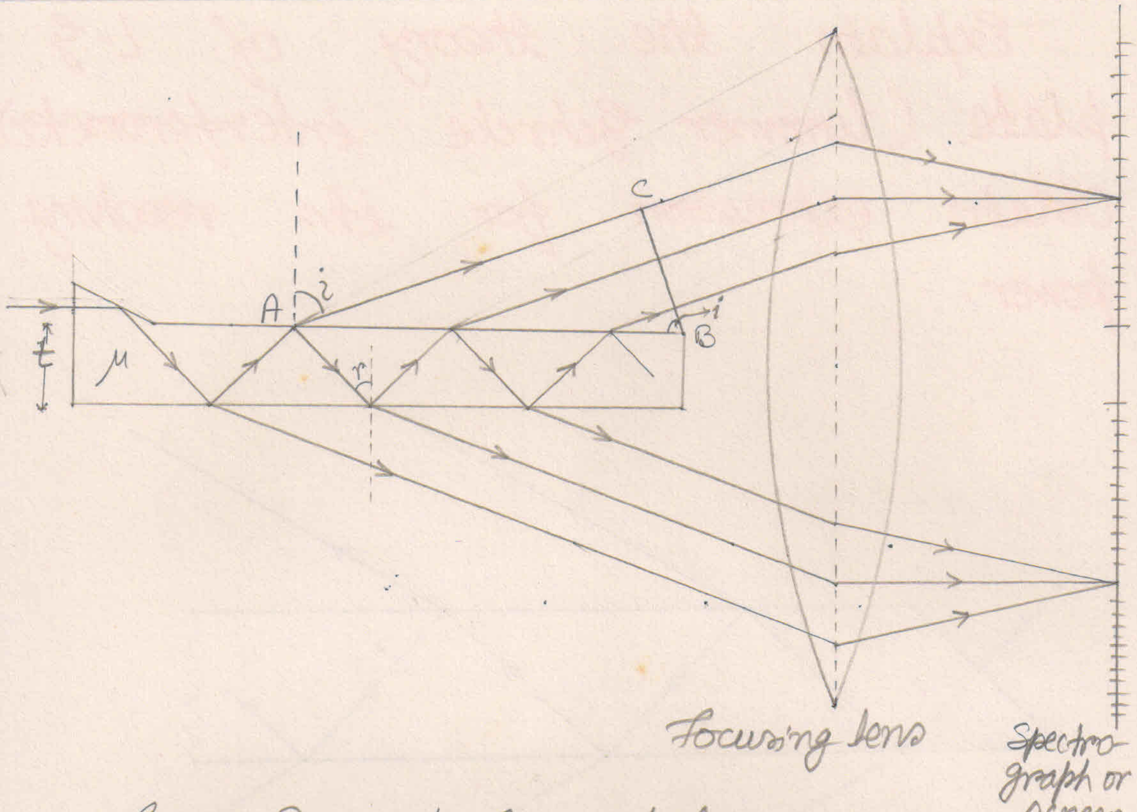


Figure 2: L. G. plate practicaly used.

AB = l = Effective length of the L. G. plate.

t = Thickness of the plate

μ = Refractive index

Focusing lens for all the rays.

The L. G. (Lummer- Gehrcke) interferometer is made of a thin flint glass plate or quartz plate. The former is useful for visible light, while the later is for ultraviolet. Irrespective of the nature of plate the incident beam is allowed to fall normally as in Fig(a) on the slanting face- or on totally reflecting plane as in Fig (1a). The later arrangement is advantegeous and practicaly used. Therefore theory is developed for this arrangement.

As shown in Fig 2, if r = angle of reflection within the plate, then condition for maximum of interference of the beams is defined by

$$2\mu \cdot t \cos r = n\lambda \quad \ldots\ldots (i)$$

where n is any integer, λ is the wavelength.

According to Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \cos r = \frac{\sqrt{\mu^2 - \sin^2 i}}{\mu} \quad \ldots\ldots\ldots (ii)$$

Using this result in equation (i) we obtain

$$2t \cdot \sqrt{\mu^2 - \sin^2 i} = n\lambda \quad \ldots\ldots\ldots (iii)$$

On squaring both sides, we obtain

$$4t^2(\mu^2 - \sin^2 i) = n^2\lambda^2 \quad \ldots\ldots\ldots (iv)$$

Different properties of the interferometer or L-G plate are explained with expression (iv)

- **Separation of orders :**

This property is defined by angle Δi between successive fringes of monochromatic light (i.e., λ constant); while n varies. Therefore taking differential both sides of expression (iv) and writing the results in measurable of finite terms we obtain

$$-4t^2 \sin 2i \, \Delta i = 2\lambda^2 n \Delta n$$

In this if we put $\Delta n = 1$, then we obtain separation between successive orders of spectrum and obtain

$$\Delta i = \frac{n \cdot \lambda^2}{-2t^2 \cdot \sin 2i}$$

$$\text{or, } \Delta i = -\lambda \frac{\sqrt{\mu^2 - \sin^2 i}}{t \cdot \sin 2i} \quad \ldots\ldots \text{(iva)}$$

where results of equation(iv) has been used.

This property depends on

a) thickness t,

b) refractive index μ and

c) angle of incidence i

It does not depend on the length of the plate.

Dispersion:

In a given order of spectrum the dispersion is defined as a change of angle of incidence (angle of refraction changes) and refractive index with respect to wavelength. This property is obtained by differentiating expression (iv) with respect to λ keeping n constant. The result is

$$4t^2\left\{2\mu\cdot\frac{\partial\mu}{\partial\lambda} - \sin 2i\,\frac{\partial i}{\partial\lambda}\right\} = 2\lambda n^2$$

$$\text{or, } \frac{\partial i}{\partial\lambda} = \frac{4t^2\cdot\mu\cdot\frac{\partial\mu}{\partial\lambda} - n^2\lambda}{2t^2\sin 2i} \quad \ldots\ldots (v)$$

Substituting in this the value of n^2 from expression (iv) we obtain

$$\frac{\partial i}{\partial\lambda} = \frac{4t^2\mu\cdot\frac{\partial\mu}{\partial\lambda} - \lambda\cdot\frac{4t^2}{\lambda^2}\left(\mu^2 - \sin^2 i\right)}{2t^2\cdot\sin 2i}$$

$$\text{or, } \frac{\partial i}{\partial\lambda} = \frac{2\mu\cdot\lambda\cdot\frac{\partial\mu}{\partial\lambda} - 2\left(\mu^2 - \sin^2 i\right)}{\lambda\cdot\sin 2i} \quad \ldots (va)$$

Thus dispersion depends on the optical property only. {size of plate does not matter}.

- **Useful range without overlap of spectrum (of different orders and different wavelengths)**

This property is obtained when we equate Δi (separation of orders of spectrum) and dispersion ∂i (angular separation of different wavelengths in one order).

Combining the results of equation (iva) and (va) with the condition $\Delta i = \partial i$, we obtain

$$\frac{\left\{4t^2\mu \cdot \frac{\partial\mu}{\partial\lambda} - n^2\lambda\right\}d\lambda}{2t^2 \cdot \sin 2i} = \frac{-n\lambda^2}{2t^2 \cdot \sin 2i}$$

This gives

$$d\lambda = \Delta\lambda = \frac{n\lambda^2}{n^2\lambda - 4t^2\mu \cdot \frac{\partial\mu}{\partial\lambda}} \quad \cdots\cdots (vi)$$

This depends on optical property μ and thickness t of the plate.

- **Expression for resolving power of L-G plate:**

The expression for resolving power of L-G plate is obtained in analogy with that of plane grating which is expressed by

$$\frac{\lambda}{d\lambda} = N \cdot m \quad \cdots\cdots\cdots (vii)$$

where N = total number of lines per cm
m = order of spectrum.

In this case we have to calculate total number of emergent rays within effective length AB Fig 2 of the L-G. plate. This is calculated with the help of Fig 3.

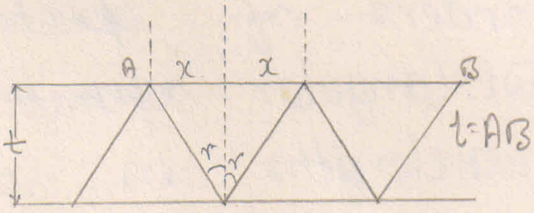


Fig 3

Let $x+x=2x$ = distance along the length of the plate covered by succesive reflected rays. From this fig we have

$$\tan r = \frac{x}{t}$$

$$\therefore 2x = 2t \cdot \tan r \quad \ldots\ldots\ldots\ldots (viii)$$

Now $2x$ length gives 1 reflected rays
Thus 1 cm gives $\frac{1}{2x}$ reflected rays
Hence l cm gives $\frac{l}{2x} = N$ beams .....(ix)

Substituting value of $2x$ in this we have

$$N = \frac{l}{2t \cdot \tan r} \quad \ldots\ldots\ldots (x)$$

Substituting this value in eqn (vii) we have

$$\frac{\lambda}{d\lambda} = N \cdot \frac{l}{2t \cdot \tan r} \quad \ldots\ldots (xi)$$

As may be seen this property depends on the size of the plate and angle of incidence on the plate.