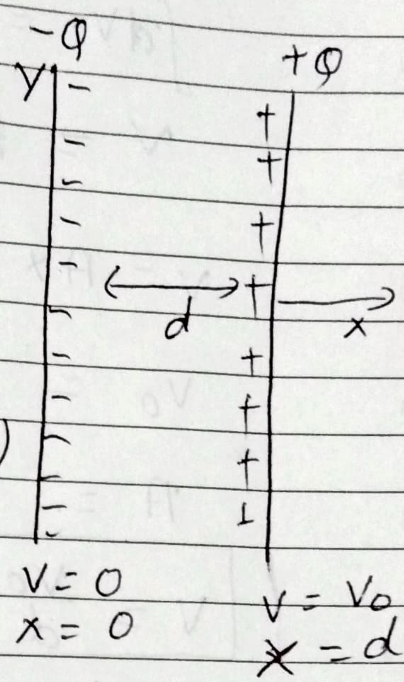
LAPLACE'S EQUATION IN RECTANGULAR CO-ORDINATED

Second order partial difference equation is known as Laplaces equation



Cartesian solution in one dimension
(field b/w two Parallel plate)

Laplaces equation calculate the potential distribution between the plate.

No variation in y and z direction and Laplace's equation

$$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{d^2 V}{\partial x^2} = 0$$

Partial derivative replace by ordinary derivative as V is not function of y and z

$$\frac{\partial^2 V}{\partial x^2} = 0 \qquad \int \frac{d^2 V}{dx^2} = \int 0$$

$$\frac{dV}{dx} = \text{const.} = A$$

$$\int \frac{dV}{dx} = \int A$$

$$\int dV = \int A\,dx$$

$$V = Ax + B$$

Boundary condition
$V = 0 \quad x = 0$
$V = V_0 \quad x = d$

$$V = Ax \qquad B = 0$$

$$V_0 = Ad$$

$$A = \frac{V_0}{d}$$

$$\boxed{V = \frac{V_0}{d}\, x \text{ volts}}$$

Another boundary condition
$V = V_1 \quad x = d_1$
$V = V_2 \quad x = d_2$

$V_1 = Ad_1 + B$ and $V_2 = Ad_2 + B$

Substraction

$$V_1 - V_2 = A(d_1 - d_2)$$

$$A = \frac{V_1 - V_2}{d_1 - d_2}$$

Sunday 27 Multiply this equation by d_2 and d_1 and sub

$$V_1 d_2 = A d_1 d_2 + B d_2$$

and $V_2 d_1 = A d_2 d_1 + B d_1$

$$V_1 d_2 - V_2 d_1 = B(d_2 - d_1)$$

$$\beta = \frac{V_1 d_2 - V_2 d_1}{d_2 - d_1}$$

$$\boxed{V = \frac{V_1 - V_2}{d_1 - d_2} x + \frac{V_1 d_2 - V_2 d_1}{d_2 - d_1}}$$

— x —