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B.Sc Sem II

Reflection and transmission of energy

When a wave meets a boundary between two media having different impedance values a part of it is reflected and a part transmitted. If A_1 is the amplitude of the incident wave in the medium of impedance Z_1 (linear density ρ_1, wave velocity v_1). B_1 the amplitude of the transmitted wave. A_2 the amplitude of the transmitted wave in the medium of impedance Z_2 (linear density ρ_2) wave velocity v_2), then

The reflection coefficient

$$\frac{B_1}{A_1} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \quad \text{—— (1)}$$

and transmission coefficient

$$\frac{A_2}{A_1} = \frac{2Z_1}{Z_1 + Z_2} \quad \text{—— (2)}$$

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In a general case, the total energy per unit length of a string of linear density ρ amplitude A and angular frequency ω is given by

$$E = \frac{1}{2}\rho\omega^2 A^2$$

As the wave travels along the string with a velocity v each unit length of the string takes up its ~~osc~~ oscillation with passage of the wave.

$\therefore$ Rate of flow of energy along the string = Energy × Velocity = $\frac{1}{2}\rho\omega^2 A^2 v$

If the point at which reflaction takes place is at $x=0$, then the rate of energy arriving at the boundary $x=0$, with the incident wave. $= \frac{1}{2}\rho_1\omega^2 A_1^2 V_1 = \frac{1}{2}Z_1\omega^2 A_1^2$

$\because (\rho_1 V_1 = Z_1)$

The energy ~~travel~~ leaves the boundary via the reflected and the transmitted waves.

Therefore rate at which energy leaves the boundary

$$= \frac{1}{2}\rho_1 \omega^2 B_1^2 v_1 + \frac{1}{2}\rho_2 \omega^2 A_2^2 v_2$$

$$= \frac{1}{2} Z_1 \omega^2 B_1^2 + \frac{1}{2} Z_2 \omega^2 A_2^2$$

Substituting the values of B_1 and A_2 from (i) and (ii) the above expression.

$$= \frac{1}{2}\omega^2 A_1^2 \frac{Z_1(Z_1 - Z_2)^2 + 4Z_1^2 Z_2}{(Z_1 + Z_2)^2}$$

$$= \frac{1}{2} Z_1 \omega^2 A_1^2$$

which is equal to the energy arriving with the incident wave.

All the energy arriving at the boundary in the incident wave leaves the boundary in the reflected and transmitted wave.