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Wave Function of Stationary Waves.

The equation of a progressive wave is

$y = a \sin \frac{2\pi}{\lambda}(vt - x)$, where λ is the wavelength & v is the velocity of the wave.

The velocity of stationary wave is

$$y = a \sin \frac{2\pi}{\lambda} vt . \cos \frac{2\pi}{\lambda} x .$$

Thus, the wave function of progressive wave is composite function of x and t. i.e, $y = f(x,t)$. On the other hand that for standing wave it is product of two separate function of x and t.

i.e, $y = f_1(x), f_2(t)$

* STATIONARY WAVES IN A CLOSED ORGAN PIPE

stationary waves are produced in closed organ pipe due to the superposition of the direct waves and the waves reflected from the closed end.

The analytical expression for the direct wave is given by

$$Y_1 = a \sin \frac{2\pi}{\lambda}(vt - x)$$

The closed end of the organ pipe must remain a node hence on reflection the amplitude must change from +a to -a and the wave must travel in the backward direction, i.e, along the negative direction of x - axis.

Equation of the reflected wave is given by

$$Y_2 = -a \sin \frac{2\pi}{\lambda}(vt + x)$$

or,

The resultant displacement y at the same time is given by

$$y = Y_1 + Y_2$$

or, $$y = a \sin \frac{2\pi}{\lambda}(vt - x) - a \sin \frac{2\pi}{\lambda}(vt + x)$$

or, $$y = -2a \cos \frac{2\pi}{\lambda} vt \sin \frac{2\pi}{\lambda} x \quad \text{(1)}$$

$$A = -2a \sin \frac{2\pi}{\lambda} x$$

$$\frac{dy}{dt} = \frac{4\pi a v}{\lambda} \sin \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda} \quad \text{(2)}$$

and acceleration

$$\frac{d^2y}{dt^2} = \frac{8\pi^2 a v^2}{\lambda^2} \sin\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda} \quad \text{---} (3)$$

$$\frac{dy}{dx} = -\frac{4\pi a}{\lambda} \cos\frac{2\pi x}{\lambda} \cos\frac{2\pi v t}{\lambda} \quad \text{---} (4)$$

Position of Nodes :-

$$\sin\frac{2\pi}{\lambda}x = 0 \quad \& \quad \cos\frac{2\pi}{\lambda}x = \pm 1$$

from eqn (1) we have $y = 0$ & $A = 0$

$$\therefore \frac{dy}{dt} = 0 \quad \& \quad \frac{d^2y}{dt^2} = 0$$

$$\frac{dy}{dx} = \mp \frac{4\pi a}{\lambda} \cos\frac{2\pi y t}{\lambda}$$

$$\sin\frac{2\pi x}{\lambda} = 0$$

$\sin m\pi = 0$ where $m = 0, 1, 2, \ldots$ etc

$$\therefore \frac{2\pi x}{\lambda} = m\pi \quad \text{or} \quad x = m\frac{\pi}{2}$$

Hence $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \ldots$ etc.

Thus, the nodes are equidistant & are separated by $\frac{\lambda}{2}$. At $x = 0$, the pasion of closed end is therefore, a node.

Position of antinodes:-

If $\sin \frac{2\pi x}{\lambda} = \pm$ & $\cos \frac{2\pi x}{\lambda} = 0$

Displacement $y = \pm 2a \cos \frac{2\pi \nu t}{\lambda}$

Amplitude $A = \pm 2a$

velocity $\frac{dy}{dt} = \pm \frac{4\pi a v}{\lambda} \sin \frac{2\pi v t}{\lambda}$

Acceleration $\frac{d^2y}{dt^2} = \mp \frac{8\pi^2 a \nu^2}{\lambda^2} \cos \frac{2\pi \nu t}{\lambda}$

and Strain $\frac{dy}{dx} = 0$

Such points where the amplitude is maximum and strain is zero and called **antinodes.**

$\because \sin \frac{2\pi x}{\lambda} = \pm 1$

But $\sin \frac{(2m+1)\lambda}{2} = \pm 1$ where $m = 0, 1, 2, \ldots$ etc

$\therefore \frac{2\pi x}{\lambda} = \frac{(2m+1)\pi}{2}$ or $x = \frac{(2m+1)\lambda}{4}$

or, $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$ etc

Thus, we find that the antinodes are also equidistant and separated by $\frac{\lambda}{2}$. The first antinode is at a distance $\frac{\lambda}{4}$ from the closed end of the organ pipe & second antinode at a distance $\frac{3\lambda}{4}$, Hence an antinode lies between two consecutive nodes.