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# Linear Differential Equation of Second order

The general form of linear differential equation of second order is

$$y'' + Py' + Qy = R$$ where

$P, Q, R$ are functions of $x$ only.

Let $y = vz$

$$\Rightarrow y'' Py' + Qy = R$$

becomes $v(z'' + Pz' + Qz) + v'(2z' + Pz) + v''z = R$

If $z$ is a integral included in the complementary function, then

$$z'' + Pz' + Qz = 0$$

then $v'' + v' \frac{(2z' + Pz)}{z} = \frac{R}{z}$

which can be salved for $v'$ and hence for $v$.

## Homogeneous Linear Differential Equation

A homogeneous linear differential equation of order $n$ is the form of $x^n y^{(n)} + x^{n-1} P_1 y^{(n-1)} + x^{n-2} P_2 y^{(n-2)} + \dots + P_n y = X.$

Where $p_1, p_2, \dots, p_n$ are constants and $X$ is a function of $x$. This can be transformed into a linear equation with constant co-efficient & solved by using the flowoing ~~way~~ method.

Let $z = \ln x$

$$\Rightarrow x = e^z$$

$$\therefore \frac{d}{dx} = \frac{1}{x}\frac{d}{dz}$$

$$\therefore x\frac{dy}{dx} = \frac{dy}{dz} = Dz$$

Where $D = \dfrac{d}{dz}$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{dz} + \frac{1}{x}\frac{d^2y}{dz^2}\frac{dz}{dx}$$

$$= \frac{1}{x^2}\left(\frac{d^2y}{dz^2} - \frac{dy}{dz}\right)$$

So, $x^2\dfrac{d^2y}{dx^2} = D(D-1)y$ etc.