

POYNTING THEOREM →

To find the power in a uniform plane wave in 1884 John H. Poynting developed a theorem for electromagnetic field using Maxwell's equations known as Poynting theorem.

Statement →

In a uniform plane wave at any point, product of electric field intensity \vec{E} & magnetic field intensity \vec{H} is a measure of rate of energy flow at the point.

In Mathematical form

$$\vec{P} = \vec{E} \times \vec{H}, \quad \vec{P} = \text{Poynting vector.}$$

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = - \int_V (\vec{E} \cdot \vec{J}) dV - \frac{\partial}{\partial t} \int_V \left(\frac{\epsilon \vec{E}^2}{2} + \frac{\mu \vec{H}^2}{2} \right) dV$$

Rate of energy flow power loss rate of decrease of stored energy.

From Maxwell's 3rd and 4th equation →

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \mathcal{D}}{\partial t} \quad \text{--- (1)}$$

$$\nabla \times \vec{E} = - \frac{\partial \mathcal{B}}{\partial t} \quad \text{--- (2)}$$

Now taking dot product with \vec{E} in (1) equation.

2010	OCTOBER				2010	NOVEMBER				
Mon	4	11	18	25	Mon	1	8	15	22	29
Tue	5	12	19	26	Tue	2	9	16	23	30
Wed	6	13	20	27	Wed	3	10	17	24	
Thu	7	14	21	28	Thu	4	11	18	25	
Fri	1	8	15	22	Fri	5	12	19	26	
Sat	2	9	16	23	Sat	6	13	20	27	
Sun	3	10	17	24	Sun	7	14	21	28	

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{J} \cdot \vec{E} + \epsilon_0 \frac{\partial \vec{E} \cdot \vec{E}}{\partial t} \quad (3)$$

Using vector identity:

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H})$$

Put value in equation (3)

$$\vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \epsilon_0 \frac{\partial \vec{E} \cdot \vec{E}}{\partial t}$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{J} \cdot \vec{E} - \epsilon_0 \frac{\partial \vec{E} \cdot \vec{E}}{\partial t} \quad (4)$$

Now take dot product with \vec{H} in equation (4)

$$\vec{H} \cdot (\nabla \times \vec{E}) = - \left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \right) - \vec{J} \cdot \vec{H} - \epsilon_0 \frac{\partial \vec{E} \cdot \vec{E}}{\partial t}$$

Put this value in eqn (3)

$$\nabla \cdot (\vec{E} \times \vec{H}) = - \left(\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \right) - \vec{J} \cdot \vec{H} - \left(\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \right) \epsilon_0 \quad (5)$$

Let us consider the term

$$\frac{\partial}{\partial t} (\vec{H} \cdot \vec{H}) = \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$$

$$\frac{\partial}{\partial t} (H^2) = 2 \vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} H^2$$

2019	AUGUST					SEPTEMBER				
Mon	30	2	9	16	23	Tue	4	11	18	25
Tue	31	3	10	17	24	Wed	5	12	19	26
Wed		4	11	18	25	Thu	6	13	20	27
Thu		5	12	19	26	Fri	7	14	21	28
Fri		6	13	20	27	Sat	8	15	22	29
Sat		7	14	21	28	Sun	9	16	23	30

Similarly, $\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \frac{\partial E^2}{\partial t}$

Put these values in equation (6).

$$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = -\frac{\mu}{2} \frac{\partial H^2}{\partial t} - \vec{E} \cdot \vec{J} - \frac{\epsilon}{2} \frac{\partial E^2}{\partial t}$$

Taking volume integral on both sides

$$\int_V \vec{\nabla} \cdot (\vec{E} \times \vec{H}) dV = -\frac{\mu}{2} \int_V \frac{\partial H^2}{\partial t} dV - \int_V (\vec{E} \cdot \vec{J}) dV - \frac{\epsilon}{2} \int_V \frac{\partial E^2}{\partial t} dV$$

Applying Gauss divergence theorem

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_V \frac{\mu H^2}{2} dV - \int_V (\vec{E} \cdot \vec{J}) dV - \frac{\partial}{\partial t} \int_V \frac{\epsilon E^2}{2} dV$$

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = -\int_V (\vec{E} \cdot \vec{J}) dV - \frac{\partial}{\partial t} \int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2} \right) dV$$