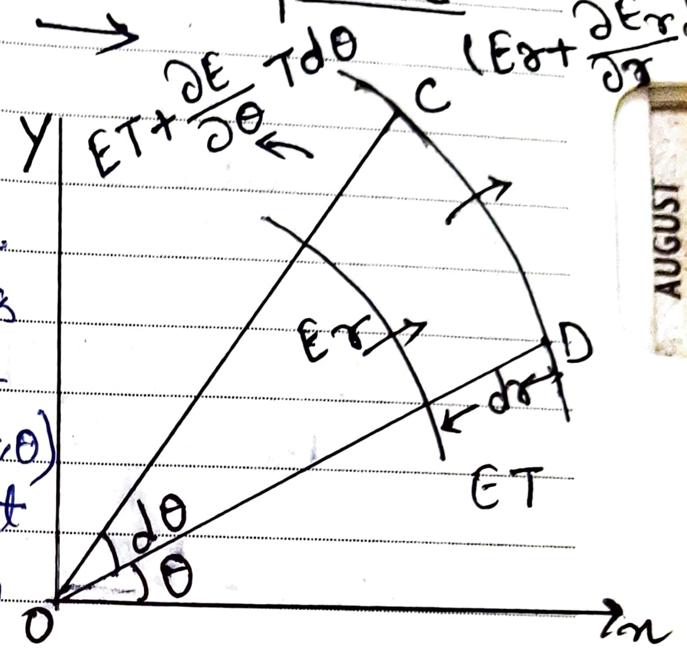
Que:- Laplace's Equation In polar co-ordinates →



Ans:- Let E_r and E_T be the radial and the tangential components of electric intensity at a point (r,θ). The radial component of fields are shown on AB and CD and tangential components on AD and BC respectively as shown in fig. The flux passing through the prism ABCD of unit length is given by

$$\left[\left(E_r + \frac{\partial E_r}{\partial r} dr\right)(r+dr)\, d\theta - E_r \cdot r\, d\theta\right]$$

$$+ \left[\left(E_T + \frac{\partial E_T}{\partial \theta} d\theta\right) dr - E_T\, dr\right]$$

$$\Rightarrow r \frac{\partial E_r}{\partial r} dr\, d\theta + E_r\, dr\, d\theta + \frac{\partial E_T}{\partial \theta} dr\, d\theta$$

But $E_r = -\frac{\partial v}{\partial r}$ and $E_T = -\frac{1}{r} \cdot \frac{\partial v}{\partial \theta}$

$\therefore \frac{\partial E_r}{\partial r} = -\frac{\partial^2 v}{\partial r^2}$ and $\frac{\partial E_T}{\partial \theta} = -\frac{1}{r} \cdot \frac{\partial^2 v}{\partial \theta^2}$

$\therefore$ Total flux passing through the prism

$$\Rightarrow r\left(-\frac{\partial^2 v}{\partial r^2}\right) dr\,d\theta + \left(-\partial v\right) d\theta + \left(-\frac{1}{r}\frac{\partial^2 v}{\partial \theta^2}\right) dr\,d\theta$$

Since no actual charges are present in the prism, therefore, total flux is zero.

$$\therefore -\left(r\frac{\partial^2 v}{\partial r^2} dr \cdot d\theta + \partial v\, d\theta + \frac{1}{r}\frac{\partial^2 v}{\partial \theta^2} dr\,d\theta\right) = 0$$

Dividing by $dr\,d\theta$, we get

$$r\frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial^2 v}{\partial \theta^2} = 0$$

Dividing by r again, we have

Notations

$$\Rightarrow \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0 \quad \text{—— (i)}$$

This is the required laplace's equation.

The End