i.e if $A = [0,1) \cup [2,3]$ subspace of X

then the map $q : A \longrightarrow Y$

where q is restriction of p on A

i.e $q = p/_A$ $\left(p(a) = q(a) \ \forall\, a \in A \right.$

then q is cts & surjective

for the set $[2,3]$ is open in A and is saturated w.r.t. q but

$$q([2,3]) = [1,2] \text{ not open in } Y.$$

*

Ex(2) Let $\pi_1 : \mathbb{R}\times\mathbb{R} \longrightarrow \mathbb{R}$ be a projection onto the first co-ordinate.

then π_1 is continuous.

Let $A \subseteq \mathbb{R}$ is open

then $\pi_1^{-1}(A) = A \times \mathbb{R}$ open in $\mathbb{R}\times\mathbb{R}$,

π_1 is surjective, obviously.

π_1 is open map:-

let $U \times V$ is open in $\mathbb{R}\times\mathbb{R}$

$\pi_1(U \times V) = U$ open in $\mathbb{R}$

$\therefore \pi_1$ is cts, open and surjective map

$\Rightarrow \pi_1$ is quotient map

π_1 is not closed map:-

Let $S = \{x \times y : xy = 1\} \subseteq \mathbb{R}\times\mathbb{R}$,

then S is closed subset of $\mathbb{R}\times\mathbb{R}$.

$\therefore \pi_1(S) = \mathbb{R}\setminus\{0\}$ which is not closed in $\mathbb{R}$

Note that if $A = S \cup \overbrace{\{0 \times 0\}}^{\{0\}} \subseteq \mathbb{R}\times\mathbb{R}$

then $q : A \longrightarrow \mathbb{R}$ formed by restricting