2.2. "NORMED SPACE. BANACH SPACE":-

NORM :-

Let X be a vector space over a field $\mathbb{K}$. A norm on a vector space X (real or complex) is a real valued function on X. $\|\cdot\| : X \longrightarrow \mathbb{R}$ whose value at an $x \in X$ is denoted by $\|x\|$ and which satisfies the following properties :-

(i) $\|x\| \geq 0$ (non-negativity)

(ii) $\|x\| = 0 \iff x = 0$

(iii) $\|\alpha x\| = |\alpha|\|x\|$ (homogenity of norm)

(iv) $\|x+y\| \leq \|x\| + \|y\|$ (Triangle inequality)

for all $x, y \in X$ and $\alpha \in \mathbb{K}$.

* A norm on X defines a metric d on X which is given by

$$d(x,y) = \|x-y\| \; ; x, y \in X,$$

and is called the metric induced (or generated) by the norm

NORMED SPACE :-

A vector space X with a norm $\|\cdot\|$ defined on it is called normed space and is denoted by ~~(X,d)~~ $(X, \|\cdot\|)$ or simply X. It is also called a normed vector space or normed linear space.

→ A complete normed space (complete in the metric defined by the norm) is called the BANACH SPACE

* Every normed space and Banach space is a metric space but converse may not hold.

* A normed space is said to be real or complex depending on the field.