* The norm is continuous. That is $x \longmapsto \|x\|$ is a continuous mapping of $(X, \|\cdot\|)$ into $\mathbb{R}$.

Examples:

(1) Let $X = \mathbb{R}$.

Define $\|\cdot\|: X \longrightarrow \mathbb{R}$ as

$$\|x\| = |x| \quad \forall x \in X.$$

$(\mathbb{R}, \|\cdot\|)$ is normed space (and a Banach space also).

(2) Euclidean space $\mathbb{R}^n$.

Let $X = \mathbb{R}^n$.

(1) Define $\|\cdot\|_2: X \longrightarrow \mathbb{R}$ by

$$\|x\|_2 = \left(\sum_{i=1}^{n} |x_i|^2\right)^{\frac{1}{2}} \quad \forall x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n.$$

$(\mathbb{R}^n, \|\cdot\|_2)$ is a Banach space.

(3) Unitary space $\mathbb{C}^n$.

Let $X = \mathbb{C}^n$.

Define $\|\cdot\|_2: X \longrightarrow \mathbb{R}$ by

$$\|x\|_2 = \left(\sum_{i=1}^{n} |\xi_i|^2\right)^{\frac{1}{2}} \quad \forall x = (\xi_1, \xi_2, \ldots, \xi_n) \in \mathbb{C}^n$$

$(\mathbb{C}^n, \|\cdot\|_2)$ is a Banach space.

4(a). Define $\|\cdot\|_\infty: \mathbb{R}^n \longrightarrow \mathbb{R}$ by

$$\|x\|_\infty = \sup_{1 \le i \le n} |x_i| \quad \forall x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$$

OR,

$$\|x\|_\infty = \max_{1 \le i \le n} |x_i| \quad \forall x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$$

This is called the sup-norm (or the uniform norm)