Norm of $\mathbb{R}^n$

① 2-norm

② l^1-norm

③ p-norm.

② (b) Define $\|\cdot\|_1 : \mathbb{R}^n \longrightarrow \mathbb{R}$ by

$$\|x\|_1 = \left(\sum_{i=1}^{n} |\xi_i|^1\right)^{\frac{1}{1}} \quad \forall,\ x = (\xi_1, \xi_2, \dots, \xi_n) \in \mathbb{R}^n.$$

$\|\cdot\|_1$ is called l^1-norm and $(\mathbb{R}^n, \|\cdot\|_1)$ is a Banach space.

③ (c) Define $\|\cdot\|_p : \mathbb{R}^n \longrightarrow \mathbb{R}$ by

$$\|x\|_p = \left(\sum_{i=1}^{n} |\xi_i|^p\right)^{\frac{1}{p}} \quad \forall\ x = (\xi_1, \xi_2, \dots, \xi_n) \in \mathbb{R}^n.$$

for, $\underline{1 \le p < \infty}$, $\|\cdot\|_p$ is called the p-norm.

$\longrightarrow$ If $\underline{0 < p < 1}$; then $\|\cdot\|_p$ is not a norm on $X = \mathbb{R}^n$

<u>Counter example.</u>

Let $x = (1, 0, 0, \dots, 0) \in \mathbb{R}^n$

$y = (0, 1, 0, \dots, 0) \in \mathbb{R}^n$.

then

(i) $\|x\|_p \ge 0$

(ii) $\|x\|_p = 0 \Leftrightarrow x = 0$

(iii) $\|\alpha x\|_p = |\alpha| \|x\|_p$

} satisfied.

but (iv) $\|x\|_p = 1^p + 0^p + \dots + 0^p = 1$, $\|y\|_p = 0^p + 1^p + \dots + 0^p = 1$

& $\|x+y\|_p = (1^p + 1^p + 0^p + \dots + 0^p)^{1/p} = (2 \cdot 1^p)^{\frac{1}{p}} = 2^{1/p} \quad \because p < 1 \ \therefore \frac{1}{p} > 1$

so, we get, $\|x+y\|_p \not\le \|x\|_p + \|y\|_p$ if $0 < p < 1$ { triangle equality not hold

$\|x+y\|_p > \|x\|_p + \|y\|_p$ if $0 < p < 1$.

$\Rightarrow \|\cdot\|_p$ is not a norm on X if $0 < p < 1$.

(5) <u>The sequence space l^p</u> :-

Let $p \ge 1$ be fixed real number.

Each element in the space l^p is a sequence $x = \langle \xi_i \rangle_{i=1}^{\infty} = \langle \xi_1, \xi_2, \dots \rangle$ of numbers s.t the sereces $\sum_{i=1}^{\infty} |\xi_i|^p$ cgt. or $\sum_{i=1}^{\infty} |\xi_i|^p < \infty$

i.e,

$$l^p = \left\{ x = \langle \xi_i \rangle_{i=1}^{\infty} \;\middle|\; \sum_{i=1}^{\infty} |\xi_i|^p < \infty,\ p \ge 1 \text{ fixed} \right\}.$$

l^p is a vector space under pointwise addition and multiplication.