Define $\|\cdot\|_p : \ell^p \longrightarrow \mathbb{R}$ by
$$\|x\|_p = \left(\sum_{i=1}^{\infty}|\xi_i|^p\right)^{\frac{1}{p}} \quad \forall\ x = \langle \xi_i \rangle_{i=1}^{\infty} \in \ell^p, \quad 1 \le p < \infty.$$
$(\ell^p, \|\cdot\|_p)$ is a normed space (& also a Banach sp.)

Eg: Let $x = \langle \xi_i \rangle_{i=1}^{\infty} = \langle \frac{1}{n} \rangle_{n=1}^{\infty} \in \ell^p$ for $p>1$

then $\sum_{i=1}^{\infty} |\xi_i|^p = \sum_{i=1}^{\infty} \left|\frac{1}{i^p}\right| < \infty$ by P-Test viz says, $\sum \frac{1}{n^p}$ cgs if $p>1$, $\sum$ " dgs if $p \le 1$

ℓ^p for $p=1,2$

$$\ell^1 = \left\{ \langle \xi_i \rangle_{i=1}^{\infty} ; \sum_{i=1}^{\infty} |\xi_i| \text{ is } \text{~~dgt~~ cgt} \right\}$$

$$\ell^2 = \left\{ \{\xi_i\}_{i=1}^{\infty} ; \sum_{i=1}^{\infty} |\xi_i|^2 \text{ is cgt} \right\}$$

→ Give an example of sequence which is in $\ell^p, p>1$ i.e, in $\ell^2, \ell^3, \ldots\ldots, \ell^p, \ldots$ but not in ℓ^1.

eg: Let $x = \langle \xi_i \rangle_{i=1}^{\infty} = \langle \frac{1}{n} \rangle_{n=1}^{\infty}$ $\therefore \sum_{i=1}^{\infty} \left|\frac{1}{i}\right|^p = \sum \frac{1}{i} \sum \frac{1}{i^p} < \infty$ for $p>1$.

$$\Rightarrow x = \langle \tfrac{1}{n} \rangle_{n=1}^{\infty} \in \ell^p, \ p>1$$

but $x = \langle \xi_i \rangle_{i=1}^{\infty} = \langle \frac{1}{n} \rangle_{n=1}^{\infty}$ i.e. $\sum_{i=1}^{\infty} \left|\frac{1}{i}\right|^p = \sum \frac{1}{i}$ dgt, $p=1$.

$$\Rightarrow x = \langle \tfrac{1}{n} \rangle_{n=1}^{\infty} \notin \ell^1.$$

(6) $\ell^\infty = \left\{ \langle \xi_n \rangle_{n=1}^{\infty} : \xi_n \in \mathbb{K} \ (=\mathbb{R}) \ \forall\ n \in \mathbb{N}, \langle \xi_n \rangle \text{ is bdd} \right\}$
i.e, ℓ^∞ is the set of all real bdd sequences of ~~numbers~~ numbers.

Define $\|\cdot\|_\infty : \ell^\infty \longrightarrow \mathbb{R}$ by
$$\|x\|_\infty = \sup_{1 \le j < \infty} |\xi_j| \quad \forall\ x \in \ell^\infty \ \left(= \langle \xi_j \rangle_{j=1}^{\infty}\right).$$