$c_{00} = \{ \langle \xi_1, \xi_2, \ldots, \xi_n, 0, 0, 0, \ldots \rangle \}$

(9) The sequence space c_{00} or Φ :-

$$\Phi = \{ x = \langle \xi_j \rangle_{j\geq 1}^{\infty} : \xi_j = 0 \text{ except finitely many terms} \}$$

Define $\|\cdot\|_\infty : \Phi \longrightarrow \mathbb{R}$ by

$$\|x\|_\infty = \sup_{1\leq j<\infty} |\xi_j| \quad \forall\ x = \langle \xi_j \rangle_{j\geq 1}^{\infty} \in \Phi.$$

$(\Phi, \|\cdot\|_\infty)$ is a normed space, $\boxed{\Phi \subset c_0 \subset c \subset l^\infty}$

$c_{00} \subset c_0 \subset c \subset l^\infty$

(10) The function space $C[a,b]$:-

It is the set of all real valued cts function defined on a given closed interval $J = [a,b]$.

i.e, $C[a,b] = \{ f \mid f : [a,b] \longrightarrow \mathbb{R} \text{ is cts} \}$.

Every point (element) of $C[a,b]$ is a cts function of an ~~indeterminate~~ independent real variable $t \in [a,b]$.

Define a norm function $\|\cdot\|_\infty : C[a,b] \longrightarrow \mathbb{R}$ by

$$\|x\|_\infty = \sup_{t\in J} |x(t)| = \sup \{ x(t) : t \in J = [a,b] \}$$

Where $\forall\ x$ is a function of $t \in C[a,b]$

OR

$$\|x\|_\infty = \max_{t\in J} |x(t)|.$$

verify that $\|\cdot\|_\infty$ is a norm on $C[a,b]$.

(i) $\|x\|_\infty \geq 0 \quad \because\ |x(t)| \geq 0 \quad \forall\ t \in [a,b]$.

$$\Rightarrow \sup_{t\in J} |x(t)| \geq 0$$

$$\Rightarrow \|x\|_\infty \geq 0.$$

(ii) $\|x\|_\infty = 0 \iff \sup_{t\in J} |x(t)| = 0$

$$\iff x(t) = 0 \quad \forall\ t \in [a,b]$$

$$\iff x = 0.$$