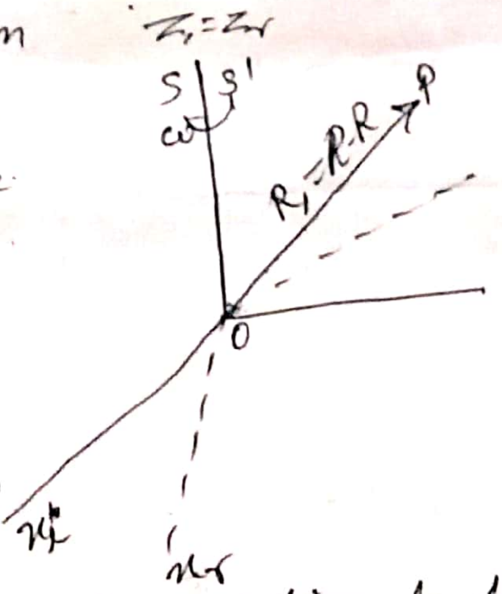
Date
24/09/2025

MDC Physics, Sem I, Unit-

Force acting on a particle in uniformly rotating frames.

The earth itself rotates about its axis in 24 hours. Therefore, any frame fixed with the earth will also rotate with it and so it will be a non-inertial frame.

Suppose that a frame $S'(x_r, Y_r, Z_r)$ is rotating with angular velocity ω relative to an inertial frame $S(x_i, Y_i, Z_i)$ for simplicity, we assume that both the frame have common origin O and common z-axis. In case of the earth, The common origin O may be considered as the center of the earth. Z-axis as coinciding with its rotational axis and the frame S'



as rotating with earth relative to the non-rotating frame S'.

The position vector of a particle of P in both frames, will be the same i.e. $R_i = R_r = R$, because the origins are coincident. Now, if the particle P is stationary in the frame S, the observer in the rotating frame S' will see that the particle is moving oppositely with linear velocity $-\omega \times R$.

Thus, if the velocity of the particle in the frame s is $\left(\frac{dR}{dt}\right)$ then its velocity, $\left(\frac{dR}{dt}\right)$, in the rotating frame will be given by

$$\left(\frac{dR}{dt}\right)_r = \left(\frac{dR}{dt}\right)_i - \omega \times R$$

$$\text{or, } \left(\frac{dR}{dt}\right)_r = \left(\frac{dR}{dt}\right)_i + \omega \times R \quad \text{—— (i)}$$

In fact this equation hold for all vectors and relates the time derivatives a vector in the frames s. Therefore ~~equation (i)~~ equation (i) may be written in the form same ~~operator~~ equation

$$\left(\frac{d}{dt}\right)_i = \left(\frac{d}{dt}\right)_r + \omega \times R \quad \text{—— (ii)}$$

writing dR/dt is equal for the velocity of the particle, we have

$$V_i = V_r + \omega \times R$$

Now, if we solve eqn (ii) on velocity V_i, we have

$$\left(\frac{dV_i}{dt}\right)_i = \left(\frac{dv_i}{dt}\right)_r + \omega \times V_i$$

Substituting the value of V_i in the right hand side of this equation from eq (ii), we obtain

$$\left(\frac{dv_i}{dt}\right)_r + \frac{d\omega}{dt} \times R + \omega\left(\frac{dR}{dt}\right)_r + \omega \times V_r + \omega \times (\omega \times R)$$

$$= \left(\frac{dV_r}{dt}\right)_r + \frac{d\omega}{dt} \times R + \omega\left(\frac{dR}{dt}\right)_r + \omega \times V_r + \omega \times (\omega \times R)$$

If we written, ~~ω is constant~~ the acceleration

$$\frac{dv}{dt} = a \text{ and } \left(\frac{dR}{dt}\right)_r = V_r \text{ then}$$

$$a_i = a_r + 2\omega \times V_r + \omega \times (\omega \times R) + \frac{d\omega}{dt} \times R$$

For earth, ω is constant, so $\frac{d\omega}{dt} = 0$, then

$$a_i = a_r + 2\omega \times V_r + \omega \times (\omega \times R) \quad \text{(iii)}$$

If m is the mass of the particle, then force in the rotating frame is

$$ma_r = ma_i \; 2m\omega \times V_r + \omega \times (\omega \times R) \quad \text{(iv)}$$

But $ma_r = F_i + F_0$ therefore ~~firi~~ fictitious force ~~in the rotating frame is f~~ F_0

F_0 is given by

$$F_0 = -2m\omega \times V_r - \omega \times (\omega \times R) \quad \text{(v)}$$

Where $-2m\omega \times V_r$ is the coriolis force and $m\omega \times (\omega \times R)$ the centrifugal force.

The Centrifugal force is the only fictitious force, acting on a particle which is at rest ($V_i = 0$) in the rotating frame. The centrifugal force may be written as

$$-m\omega \times (\omega \times R) = m\omega^2 r$$

Where r is the vector from the axis of the earth to the particle and normal to it, ~~because~~