$\Rightarrow \langle \xi_j^{(m)} \rangle$ is cgt in $\mathbb{R}$ ($\because \mathbb{R}$ is complete) $\forall j$

$\Rightarrow \lim_{m\to\infty} \xi_j^{(m)} = \xi_j \in \mathbb{R} \quad \forall j = 1, 2, \ldots, n.$

Using the n limits, we get $\xi_1, \xi_2, \ldots, \xi_n$

Let $x = (\xi_1, \xi_2, \ldots, \xi_n)$

Since $\xi_j \in \mathbb{R} \ \forall j = 1, 2, \ldots, n \ \therefore x \in \mathbb{R}^n$.

Now,

Letting $k \to \infty$ in ①, we have,

$\lim_{k\to\infty} \|x_m - x_k\| < \epsilon \quad \forall m \geq N.$

$\Rightarrow \|x_m - \lim_{k\to\infty} x_k\| < \epsilon \quad \forall m \geq N$

$\Rightarrow \|x_m - \xi_j\| \leq \epsilon \quad \forall m \geq N.$

$\Rightarrow \left(\sum_{j=1}^{n} |\xi_j^{(m)} - \xi_j|^2\right)^{\frac{1}{2}} < \epsilon \quad \forall m \geq N.$

$\Rightarrow d(x_m, x) < \epsilon \quad \forall m \geq N.$

$\Rightarrow x_m \longrightarrow x \in \mathbb{R}^n.$

$\Rightarrow \mathbb{R}^n$ is complete.

$\Rightarrow (\mathbb{R}^n, \|\cdot\|)$ is a Banach space.

(2) The sequence space l^p $(1 \leq p < \infty)$ is complete.

$$\|x\|_p = \left(\sum_{j=1}^{\infty} |\xi_j|^p\right)^{1/p} \quad \forall x = \langle \xi_j \rangle_{j=1}^{\infty} \in l^p$$

$$d(x, y) = \|x - y\|_p = \left(\sum_{j=1}^{\infty} |\xi_j - \eta_j|^p\right)^{1/p} \quad \forall \left. \begin{array}{l} x = \langle \xi_j \rangle_{j=1}^{\infty} \\ y = \langle \eta_j \rangle_{j=1}^{\infty} \end{array} \right\} \in l^p$$

(d is metric induced by norm)

Let $\langle x_n \rangle_{n=1}^{\infty}$ be a Cauchy sequence in l^p.

Then for each $\epsilon > 0$, $\exists$ a +ve integer $N = N(\epsilon)$

s.t. $\|x_n - x_m\| < \epsilon \quad \forall n, m \geq N$; $\langle x_n \rangle_{n=1}^{\infty} = \langle \xi_j^{(n)} \rangle_{j=1}^{\infty}$

$\Rightarrow d(x_n, x_m) < \epsilon \quad \forall n, m \geq N$

$\Rightarrow \left(\sum_{j=1}^{\infty} |\xi_j^{(n)} - \xi_j^{(m)}|^p\right)^{1/p} < \epsilon \quad \forall n, m \geq N. \quad \text{——} (\alpha)$

$\Rightarrow \sum_{j=1}^{\infty} |\xi_j^{(n)} - \xi_j^{(m)}|^p < \epsilon^p \quad \forall n, m \geq N.$