$\Rightarrow |\xi_j^{(n)} - \xi_j^{(m)}|^p < \epsilon^p \quad \forall\, n,m \geq N \ \&\ \forall\, j$

$\Rightarrow |\xi_j^{(n)} - \xi_j^{(m)}| < \epsilon \quad \forall\, n,m \geq N \ \&\ \forall\, j$

$\therefore$ for fixed j, the sequence $\langle \xi_j^{(n)} \rangle_{j\geq 1}^{\infty}$ is a Cauchy sequence in $\mathbb{K}$ (where $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) both are complete

$\Rightarrow \langle \xi_j^{(n)} \rangle$ is convergent in $\mathbb{R}$ $\forall\, j \in \mathbb{N}$.

Let $\lim_{n\to\infty} \xi_j^{(n)} = \xi_j \in \mathbb{R} \quad \forall\, j \in \mathbb{N}$. — (1)

Using these infinite limits, we define

$x = \langle \xi_1, \xi_2, \text{———} \rangle$

claim :- $x \in \ell^p$.

By $(*)$ we have

$$\left(\sum_{j\geq 1}^{\infty} |\xi_j^{(n)} - \xi_j^{(m)}|^p\right)^{1/p} < \epsilon \quad \forall\, n,m \geq N$$

Letting $m \to \infty$ and using (1), we get

$$\left(\sum_{j\geq 1}^{\infty} |\xi_j^{(n)} - \xi_j|^p\right)^{1/p} < \epsilon \quad \forall\, n \geq N. \text{ — (2)}$$

$\Rightarrow \|x_n - x\|_p < \epsilon \quad \forall\, n \geq N.$

$\Rightarrow x_n - x \in \ell^p$ where $x_n - x = \langle \xi_j^{(n)} - \xi_j \rangle$

Now $\|x\|_p := \|-x\|_p = \|x_n - x - x_n\|_p$

$\leq \underbrace{\|x_n - x\|_p}_{\in \ell^p} + \underbrace{\|x_n\|_p}_{\in \ell^p}$ — (3) {by Minkowski ineq

$\therefore \sum_{j\geq 1}^{\infty} |\xi_j^{n} - \xi_j|^p < \infty \ \&\ \sum_{j\geq 1}^{\infty} |\xi_j^{(n)}|^p < \infty.$

$\therefore x_n - x \in \ell^p \Rightarrow \sum_{j\geq 1}^{\infty} |\xi_j^{(n)} - \xi_j|^p < \infty$ (cgt)

$\Rightarrow \left[\sum_{j\geq 1}^{\infty} |\xi_j^{(n)} - \xi_j|^p\right]^{1/p} < \infty$

$\text{Illy } x_n \in \ell^p \Rightarrow \left(\sum_{j\geq 1}^{\infty} |\xi_j^{(n)}|^p\right)^{1/p} < \infty$

$\Rightarrow \|x\|_p \leq \underbrace{\|x_n - x\|_p}_{(\because < \infty)} + \underbrace{\|x_n\|_p}_{(<\infty)}$

$\Rightarrow \|x\|_p < \infty$