

Date

17/10/2025

Sem III, Unit IV

Page 1

CLAUSIUS INEQUALITY

Clausius inequality $\rightarrow$ In thermodynamics the Clausius inequality derived for a Carnot cycle, and then extended to any cycle. For the Carnot cycle, we have that it's the most efficient possible cycle between two heat reservoirs, with an efficiency of one (1) minus the ratio of the temperature of the cold reservoir to that of the hot one. Any other cycle between those two reservoirs must be less than or equal to the Carnot efficiency, with an efficiency of 1 minus the heat absorbed from the cold reservoir divided by the heat taken in from the hot one. Let η_{irr} be the efficiency of any irreversible cycle and η_{rev} be the efficiency of the Carnot cycle.

Here, is reversible cycle between two reservoirs

$$\eta_{rev} = 1 - \frac{T_L}{T_H} \quad \text{--- (i)}$$

$$\eta_{irr} = 1 - \frac{Q_L}{Q_H} \quad \text{(ii)}$$

$$\boxed{1 - \frac{Q_L}{Q_H} \leq 1 - \frac{T_L}{T_H}} \quad \text{--- (iii)}$$

We can derive the Clausius inequalities for Carnot-type cycle.

$$\Rightarrow \text{Reversible cycle} \rightarrow \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

$$\Rightarrow \text{Irreversible cycle} \rightarrow$$

$$\frac{Q_H}{T_H} - \frac{Q_L}{T_L} < 0$$

Somehow, Clausius extended this to any cycle, claiming that —

$$\rightarrow \text{Reversible cycle} \rightarrow \oint \frac{\delta Q}{T} = 0$$

$$\rightarrow \text{Irreversible cycle} \rightarrow \oint \frac{\delta Q}{T} < 0$$

It is Clausius inequality that applies to any cycle.