

Note: If the greatest and least points of a set S exist then there are the supremum and infimum of S .

(iii) $\left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$

Sol: let $S = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}$; $\sup S = 1 \in S$ (greatest)

both inf & sup exist so S is bdd.

$\inf S = 0 \notin S$ (smallest)

(iv) $\left\{ \frac{(-1)^n}{n}, n \in \mathbb{N} \right\}$

Sol: let $S = \left\{ \frac{(-1)^n}{n} ; n \in \mathbb{N} \right\}$

$= \left\{ -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \dots \right\}$

$\lim_{n \rightarrow \infty} S = \lim_{n \rightarrow \infty} \frac{(-1)^n}{n} = 0 \notin S$

$\left. \begin{array}{l} \sup S = \frac{1}{2} \\ \inf S = -1 \end{array} \right\} \in S \Rightarrow S \text{ is bdd.}$
 greatest member $= \frac{1}{2}$
 smallest member $= -1$

(v) $\left\{ -2, -3, -4, -5, \dots, -\frac{n+1}{n}, \dots \right\}$

Sol: let $S = \left\{ -2, -3, -4, -5, \dots, -\frac{n+1}{n}, \dots \right\}$

$\lim_{n \rightarrow \infty} S = \lim_{n \rightarrow \infty} -\frac{n+1}{n} = \lim_{n \rightarrow \infty} -\left(1 + \frac{1}{n}\right) = -1$

$\left. \begin{array}{l} \sup S = -1 \notin S \\ \inf S = -2 \in S \end{array} \right\} \Rightarrow S \text{ is bdd}$
 Smallest member $= -2$

(vi) $\left\{ 1 + \frac{(-1)^n}{n}, n \in \mathbb{N} \right\}$

Sol: let $S = \left\{ 1 + \frac{(-1)^n}{n}, n \in \mathbb{N} \right\}$

$\lim_{n \rightarrow \infty} S = \lim_{n \rightarrow \infty} \left\{ 1 + \frac{(-1)^n}{n} \right\} = 1 + 0 = 1$