

Q(5a) Show that the greatest member of a set, in case it exists, is unique.

Sol:- let if possible, k, k' be two greatest member of ~~set~~ S (where S is a set).
Now, let $k' = \sup S$ and k is the upper bound of S .

$$\Rightarrow k' \leq k \quad \text{--- (1)}$$

Again, let $k = \sup S$ and k' is an upper bound of S .

$$\Rightarrow k \leq k' \quad \text{--- (2)}$$

from (1) & (2), $k = k'$

Hence the set S has a unique greatest member.

Q(5b) Show that the smallest member of a set, in case it exists, is unique.

Sol:- let if possible k, k' be two smallest member of S .

Now let $k' = \inf S$ and k is ~~to~~ a lower bound of S .

$$\Rightarrow k \leq k' \quad \text{--- (1)}$$

Again, let $k = \inf S$ and k' is a lower bound of S .

$$\Rightarrow k' \leq k \quad \text{--- (2)}$$

from (1) & (2) $\Rightarrow k = k'$

Hence the set S has a unique smallest member.

Q(6) Show that the smallest member of a set, if it exists, is the infimum of the set.