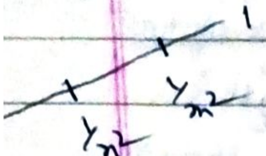
$\forall\, n > m$, we have

$$\|x_n - x_m\|_1 = \int_0^1 |x_n(t) - x_m(t)|\,dt$$

$$= \int_0^{1/n^2} |x_n(t) - x_m(t)|\,dt + \int_{1/n^2}^{1/m^2} |x_n(t) - x_m(t)|\,dt + \int_{1/m^2}^{1} |x_n(t) - x_m(t)|\,dt$$



$$= \int_0^{1/n^2} |(n-m)|\,dt + \int_{1/n^2}^{1/m^2} \left|\frac{1}{\sqrt{t}} - m\right| dt + \int_{1/m^2}^{1} \left|\frac{1}{\sqrt{t}} - \frac{1}{\sqrt{t}}\right| dt$$

$$= \int_0^{1/n^2} |n-m|\,dt + \int_{1/n^2}^{1/m^2} \frac{1}{\sqrt{t}}\,dt - \int_{1/n^2}^{1/m^2} m\,dt + 0$$

$$= \frac{n-m}{n^2} + 2\sqrt{t}\,\Big|_{1/n^2}^{1/m^2} - m\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

$$= \frac{n-m}{n^2} + 2\left[\frac{1}{m} - \frac{1}{n}\right] - m\left[\frac{1}{m^2} + \frac{1}{n^2}\right]$$

$\longrightarrow 0$ as $n, m \longrightarrow \infty$

$\Rightarrow \|x_n - x_m\|_1 \longrightarrow 0$ as $n, m \longrightarrow \infty$

$\Rightarrow \langle x_n \rangle$ is Cauchy sequence in $(C[0,1], \|\cdot\|_1)$

Let if possible $\exists\, x \in C[0,1]$ s.t. $x_n \to x$.

i.e. $\lim_{n\to\infty} x_n = x$ i.e. $\|x_n - x\|_1 \longrightarrow 0$ as $n \to \infty$

$$\Rightarrow \int_0^1 |x_n(t) - x(t)|\,dt \longrightarrow 0 \text{ as } n \to \infty$$

$$\Rightarrow \int_0^{1/n^2} |x_n(t) - x(t)|\,dt + \int_{1/n^2}^{1} |x_n(t) - x(t)|\,dt \longrightarrow 0 \text{ as } n \to \infty$$

$$\Rightarrow x(t) = \begin{cases} 0, & t = 0 \\ \frac{1}{\sqrt{t}}, & 0 < t \le 1 \end{cases}$$

$\Rightarrow x(t)$ is not continuous

$\Rightarrow x \notin C[0,1]$

$\Rightarrow (C[0,1], \|\cdot\|_1)$ is NOT Complete.

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