Problem 1

Evaluate the integral:

$$\boldsymbol{I = \int_0\text{^\{}\pi/2\text{\}}\ (sin\ x)\ /\ (1 + cos^2 x)\ dx}$$

Solution:

Substitute u = cos x ⇒ du = -sin x dx. Limits: x = 0 ⇒ u = 1, x = π/2 ⇒ u = 0.

$$I = \int_1^0 (-du)/(1 + u^2) = \int_0^1 du/(1 + u^2) = arctan(u)\ |_0^1 \\ = \pi/4$$

Answer: I = π/4.

Problem 2

Find the area bounded by $y = x^2$ and $y = 4$.

Solution:

Solve $x^2 = 4 \Rightarrow x = \pm 2$.

$$Area = \int\_\{-2\}\^\{2\}\,(4 - x^2)\,dx = [4x - x^3/3]\_\{-2\}\^\{2\}$$

$$Area = (8 - 8/3) - (-8 + 8/3) = 32/3$$

Answer: Area = 32/3 square units.

Problem 3

Find the Maclaurin series for $f(x) = e^x \cos x$ up to x^4 term.

Solution:

Using the series expansions:

$$e^x = 1 + x + x^2/2 + x^3/6 + x^4/24 + \dots$$

$$\cos x = 1 - x^2/2 + x^4/24 + \dots$$

Multiply up to x^4 and simplify to get:

$$f(x) = 1 + x - x^3/3 - x^4/6 + O(x^5)$$

Answer: $e^x \cos x = 1 + x - x^3/3 - x^4/6 + O(x^5)$.