

Solⁿ: Let s be the smallest member of a set S
 then (i) $s \in S$
 and (ii) $s \leq x, \forall x \in S$
 $\Rightarrow s$ is a lower bound of S .
 let s_1 be any number $> s$.
 then $s_1 > s$ and $s \in S$
 $\Rightarrow s_1$ can not be a lower bound of S .
 Hence s is the infimum (g.l.b) of the set S .

Example (3) prove that the greatest member of a set, if it exists, is the supremum (l.u.b) of the set.

Solⁿ: Let G be the greatest member of S .
 then (i) $G \in S$
 and (ii) $x \leq G, \forall x \in S$
 and so G is an upper bound of S .
 let G_1 be any number $< G, G_1 \in S$.
 i.e., $G_1 < G$ then G_1 can not be an upper bound of S .
 Hence G is the l.u.b (supremum) of S .

Q(7) Is the converse of the solved example (3), true?

ANS: NO, because supremum may or may not belong to the set.
 but greatest member must belong to the set.