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⑧ The normed space  $(\Phi, \|\cdot\|_\infty)$  is not complete.

(4)

$$\text{let } x_1 = \langle 1, 0, 0, \dots \rangle$$

$$x_2 = \langle 1, \frac{1}{2}, 0, \dots \rangle$$

$$x_n = \langle 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, 0, \dots \rangle$$

$$(x_n) \in \Phi$$

$$\text{let } x = \frac{1}{n}$$

$$x_n \rightarrow x \text{ as } n \rightarrow \infty$$

So  $x$  is a limit point of  $\Phi$ .

$$\Rightarrow x \in \bar{\Phi}$$

$$\text{but } x \notin \Phi$$

$\Rightarrow \Phi$  is not closed.

Also,  $\Phi \in \ell^\infty$  where  $\ell^\infty$  is Banach space.

So  $\Phi$  is not complete.

⑨ The space  $(\mathcal{D}, \|\cdot\|)$  is not complete, where  $\|\cdot\| = |\cdot|$ ;

$$\text{let } x_1 = 1$$

$$x_2 = 1 + i$$

$$x_3 = 1 + i + \frac{1}{2!}$$

$$\vdots$$

$$x_n = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{(n-1)!}$$

$$x_n \in \mathcal{D} \quad \forall n$$

As  $n \rightarrow \infty$   $x_n \rightarrow e$  but  $e \notin \mathcal{D}$ .

$\Rightarrow \mathcal{D}$  is not closed.

$\mathcal{D} \subseteq \mathbb{R}$ , where  $\mathbb{R}$  is complete.

$\Rightarrow \mathcal{D}$  is not complete.