Question 1

Evaluate $\int (x^2 + 2x + 1) e^x dx$

Solution:

We can split the integral:

$I = \int x^2 e^x dx + 2 \int x e^x dx + \int e^x dx$

We know that $\int x^n e^x dx = e^x (x^n - n x^{n-1} + n(n-1)x^{n-2} - \ldots)$

$\int x^2 e^x dx = e^x (x^2 - 2x + 2)$, $\int x e^x dx = e^x (x - 1)$, $\int e^x dx = e^x$

Therefore, $I = e^x (x^2 - 2x + 2) + 2e^x (x - 1) + e^x$

Simplify: $I = e^x (x^2 + 1) + C$

Answer: $I = e^x (x^2 + 1) + C$

Question 2

Find $\int (x^3 / (x^2 + 1))\,dx$

Solution:

Divide x^3 by $(x^2 + 1)$: $x^3 / (x^2 + 1) = x - (x / (x^2 + 1))$

So, $I = \int x\,dx - \int (x / (x^2 + 1))\,dx$

$\int x\,dx = x^2/2$

For $\int (x / (x^2 + 1))\,dx$, let $u = x^2 + 1 \Rightarrow du = 2x\,dx \Rightarrow \int (x / (x^2 + 1))\,dx = ½ \int du/u = ½ \ln|u|$

Hence, $I = x^2/2 - (1/2) \ln(x^2 + 1) + C$

Answer: $I = ½ [x^2 - \ln(x^2 + 1)] + C$

Question 3

Evaluate $\int_0^{\pi/2} \sin^3 x \, dx$

Solution:

Using the reduction formula: $\int \sin^n x \, dx = -(1/n) \sin^{n-1} x \cos x + ((n-1)/n) \int \sin^{n-2} x \, dx$

Here $n = 3 \Rightarrow I = -(1/3) \sin^2 x \cos x + (2/3) \int \sin x \, dx$

Integrate: $I = -(1/3) \sin^2 x \cos x - (2/3) \cos x + C$

Evaluate from 0 to $\pi/2$:

At $x = \pi/2$, $\sin x = 1$, $\cos x = 0 \Rightarrow$ value $= 0$

At $x = 0$, $\sin x = 0$, $\cos x = 1 \Rightarrow$ value $= -(-2/3) = 2/3$

✅ Final Answer: $I = 2/3$