$\Rightarrow \langle x_n \rangle$ Converges in Y.
The limit being unique, $x \in Y$
$\Rightarrow \bar{Y} \subset Y$
$\Rightarrow Y = \bar{Y}$
$\Rightarrow Y$ is closed.

* Absolute Convergence

Def^n: A series $\sum_{n\geq 1}^{\infty} x_n$ is said to be absolute convergent in a normed space $(X, \|\cdot\|)$, if $\sum_{n\geq 1}^{\infty} \|x_n\|$ is convergent.

→ If $\langle x_k \rangle_{k\geq 1}^{\infty}$ is a sequence in a normed space X.
we can associate with $\langle x_k \rangle_{k\geq 1}^{\infty}$ the sequence $\langle S_n \rangle$ of partial sums

$$S_n = x_1 + x_2 + \cdots + x_n \quad \text{where } n \geq 1, 2, \ldots$$

If $\langle S_n \rangle$ is convergent say $S_n \to s$
i.e. $\|S_n - s\| \to 0$
then the infinite series or briefly series.

$$\sum_{k\geq 1}^{\infty} x_k = x_1 + x_2 + \cdots$$

is said to be cgt or said to cgs. S is called the sum of the series and we write

$$S = \sum_{k\geq 1}^{\infty} x_k = x_1 + x_2 + \cdots$$

(Q.7)(1.5)
Remark: In a normed space X, an absolutely cgt series need not be cgt.
2010
Counter example
$X = \Phi$, the sequence space.
$= \{ \langle x_n \rangle_{n\geq 1}^{\infty} : x_n = 0 \text{ except finitely many terms} \}$
with $\|\cdot\|_\infty$ norm.
$(\Phi, \|\cdot\|_\infty)$ is normed space