

Poisson's equation —

Poisson's equation relates the electric potential to charge density

$$\nabla^2 V = - \frac{\rho_v}{\epsilon}$$

where V = Electric Potential
 ρ_v = volume charge density
 ϵ = dielectric constant

The potential of a conductor is proportional to the charge it contains.

Proof →

The relation between electric potential and field intensity is given as

$$E = - \nabla V \quad \text{--- (1)}$$

'E' is the gradient of electric potential in opposite direction

* Gauss's law in point form

$$\boxed{\nabla \cdot \mathbf{D} = \rho_v} \quad \text{--- (1)}$$

The divergence of electric flux density is equal to the volume charge density.

$$\nabla \cdot \mathbf{D} = \rho_v$$

$$\nabla \cdot (\epsilon \mathbf{E}) = \rho_v \quad \left(\because \mathbf{D} = \epsilon \mathbf{E} \right)$$

$$\epsilon (\nabla \cdot \mathbf{E}) = \rho_v$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_v}{\epsilon} \quad \text{--- (2)}$$

Sub. eqn (1) $\mathbf{E} = -\nabla V$ in (2)

$$\nabla \cdot (-\nabla V) = \frac{\rho_v}{\epsilon}$$

$$-\nabla \cdot \nabla V = \frac{\rho_v}{\epsilon}$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

This is called as Poisson's equation.

For cartesian co-ordinate system (x, y, z)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon}$$

For cylindrical co-ordinate system (ρ, ϕ, z)

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \left(\frac{\partial^2 V}{\partial \phi^2} \right) + \frac{\partial^2 V}{\partial z^2} = -\frac{\rho_v}{\epsilon}$$

For Spherical co-ordinate system (r, θ, ϕ)

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = -\frac{\rho_v}{\epsilon}$$

LAPLACE'S EQUATION

If an enclosed surface does not have any volume charge ρ_v , then the Poisson's equation becomes

$$\nabla^2 V = 0$$

$$\therefore \rho_v = 0$$

This is called as Laplace equation

[$\nabla^2 \rightarrow$ Laplacian operator]

$\rightarrow$ The solutions of Laplace's equation are the harmonic function

$\rightarrow$ Harmonic functions are important in partial differential equation theory, physics and engineering field.

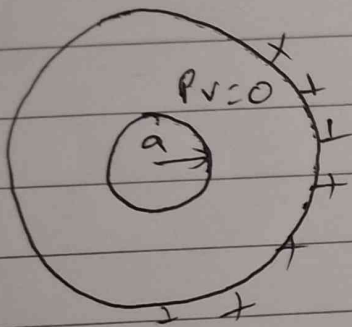
Applications $\rightarrow$

It is used in boundary value problems.

In such cases, the charge distribution is invulved on the surface of the conductor for which the volume charge density is zero.

Ex: -

To obtain potential and capacitance in between two surface.



Potential between conductors of a coaxial cable.

