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Paper-10 unit-2
MJC; Sem-IV

Uses of canonical transformations

A transformation $(q, P, t) \rightarrow (Q, P, t)$ is canonical if it maintain the structure of Hamiltonian's equations, often leaving the poission brakets $[Q_i, Q_j]_{qp} = 0$, $[P_i, P_j]_{qp} = 0$ and $[Q_i, P_j]_{qp} = \delta_{ij}$ inveriant.

Canonical transformation are often defined using generating function $F(q, Q, t)$, $F(q, P, t)$ etc. Which bridge the old and new coordinates ensuring the transformation is valid.

It's Invariant form is the new Hamiltonian K is related to the old Hamiltonian H by $K = H + \frac{\partial F}{\partial t}$. And purpose of this is →

They are used to simplify equation of motion, notably in the Hamiltonian - Jacobi theory to find conserved quantities.

Relativistic machenics →: To align Hamiltonian machenics with Special relativity, One must use a reletivistic Hamiltonian, typically $H = \sqrt{p^2c^2 + m^2c^4} + V(q)$, Which ensures the equation of motion are compatible with 4-momentum transformations.