Integral of the form: $\int (px+q)\sqrt{ax^2+bx+c}\,dx$

$$px+q = \frac{d}{dx}(ax^2+bx+c) + k$$

Q1 : $\int x\sqrt{x^2+x}\,dx$

$$\frac{d}{dx}(x^2+x) = 2x+1$$

now $\quad 1\cdot x = k_1(2x+1) + k_2$

$$k_1 = 1/2 \quad ; \quad k_1 + k_2 = 0 \Rightarrow k_2 = -1/2$$

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$$\int x\sqrt{x^2+x}\,dx = \underbrace{\int \frac{1}{2}(2x+1)\sqrt{x^2+x}\,dx}_{I_1} - \underbrace{\frac{1}{2}\int\sqrt{x^2+x}\,dx}_{I_2}$$

$$I_1 = \frac{1}{2}\int (2x+1)\sqrt{x^2+x}\,dx \quad ; \text{ let } x^2+x=t$$

$$(2x+1)dx = dt$$

$$= \frac{1}{2}\int \sqrt{t}\,dt = \frac{1}{2}\frac{t^{3/2}}{3/2} = \frac{1}{3}(x^2+x)^{3/2} + c_1$$

$$I_2 = x^2+x+\frac{1}{4}-\frac{1}{4} = \left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$-\frac{1}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}\,dx = -\frac{1}{2}\left[\frac{x+\frac{1}{2}}{2}\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2} - \frac{1}{8}\log\left|\left(x+\frac{1}{2}\right)+\sqrt{x^2+x}\right|\right.$$

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2. Evaluate: $\displaystyle\int_1^2 \frac{1}{x(1+\log x)^2}\,dx$

Soln: Let $1+\log x = t$; $1+\log 1 = t \Rightarrow t=1,\ x=1$

$\frac{1}{x}\,dx = dt$ $\qquad 1+\log 2 = t$

$$I = \int_1^2 \frac{1}{x(1+\log x)^2}\,dx = \int_1^{1+\log 2} \frac{dt}{t^2} = -\left[\frac{1}{t}\right]_1^{1+\log 2}$$

$$= -\left[\frac{1}{1+\log 2} - \frac{1}{1}\right] = \frac{\log 2}{1+\log 2} \quad \text{Ans.}$$