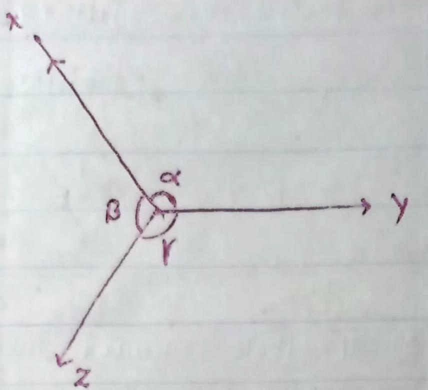
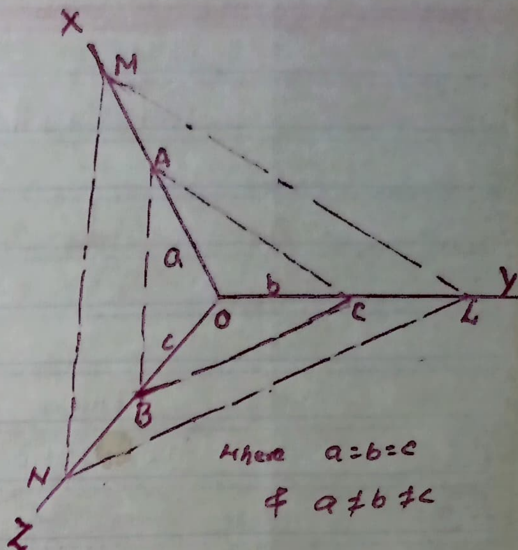
Hauy's LAW OF RATIONAL INTERCEPT.





where $a = b = c$

& $a \neq b \neq c$

Intercept $OM : OL : ON = la : mb : nc$

Where l, m, n are simple integer is 1, 2 or 3.

The law may be stated as :—

The intercept of any face of a crystal along the crystalographic axies are either equal to unit intercepts (a, b, c) or small whole number multzplier by them (unit intercept) i.e la, mb, nc, where $l, m,$ and n are simple whole numbers.

WEISS INDICES :— Ratio of intercepts of plane LMN and standard plane ABC is called weiss indices.

If the intercept of plane LMN is designated by $la ; mb, nc$ and the intercept of standard plane ABC is designated by a, b, c,

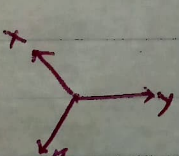
then,

$$\frac{la}{a} : \frac{mb}{b} : \frac{nc}{c}$$

$$l : m : n$$

The plane LMN is characterised or represented by (l, m, n) plane.

Other plane M'L'N' whose intercept is $2a : 2b : c$



then Ratio of intercept (weiss indices) is

$$\frac{2a}{a} : \frac{2b}{b} : \frac{c}{c}$$

$$2 : 2 : 1$$

Now this plane M'L'N' is characterised by $(2, 2, 1)$

It should be noted that the Weiss indices have been completly replaced by Miller Indices.

Weiss indices	Miller indices
l	$\frac{1}{l} \cdot x = h$
m	$\frac{1}{m} \cdot x = k$
n	$\frac{1}{n} \cdot x = l$

Where x is small whole number used to bring the ratio into whole Number.

Plane	Weiss indices	Miller indices.	Miller index Plane
(2,1,2)	2	$\frac{1}{2} \times 2 = 1$	(1,2,1)
	1	$\frac{1}{1} \times 2 = 2$	
	2	$\frac{1}{2} \times 2 = 1$	

Miller indices :→

Any Particular face of a crystal is represented by the reciprocals of the multiples of the unit intercept.

Let us consider on the unit Plane ABC, the intercepts a, b, and c are all units. therefore the reciprocals are in the ratio 1:1:1, so the Miller indices of the ABC face are (1,1,1). Now we suppose a face cuts only two axes OY and OZ, intercepts being 2 and 3 multiples of b and c. It does not cut the axis OX, but being Parallel to it. Hence the reciprocal of multiples are $0, \frac{1}{2}, \frac{1}{3}$.

Plane	Reciprocal of Multiples	Miller index
(0, 2, 3)	$0 \times 6 = 0$	0
Here 2, and 3	$\frac{1}{2} \times 6 = 3$	3
are multiples of b and c	$\frac{1}{3} \times 6 = 2$	2

∴ Miller indeces (0, 3, 2)

So, the Miller indices of any Crystal face are inversely proportional to the intercepts of that face on the different axes.

The distance between the Parallel Planes in a Crystal are designated as d_{hkl}. For different Cubic lattices, the interplaner Spacing (inter planer distance) are given by the General formula:

$$d_{h.k.l.} = \frac{a}{\sqrt{(h^2+k^2+l^2)}}$$

Where 'a' is the length of the Cube Side, While h, k and l are Miller indices of the Plane.

Problem: — How do the spacing of the three planes (100) (110) and (111) of a Cubic lattice vary.

We Know, $d_{h,k,l} = \dfrac{a}{\sqrt{h^2+k^2+l^2}}$

$$d_{100} = \frac{a}{\sqrt{1^2+0^2+0^2}} = a$$

$$d_{110} = \frac{a}{\sqrt{1^2+1^2+0}} = a/\sqrt{2}$$

$$d_{111} = \frac{a}{\sqrt{1^2+1^2+1^2}} = a/\sqrt{3}$$

$$\therefore d_{100} : d_{110} : d_{111} = a : a/\sqrt{2} : a/\sqrt{3}$$
$$= 1 : 1/\sqrt{2} : 1/\sqrt{3}$$

(i) Simple Cubic lattice.

$d_{(100)} : d_{(110)} : d_{(111)}$ $= 1 : .707 : .577$

(ii) Face Centred Cubic lattices,

$$d_{(100)} : d_{(110)} : d_{(111)} = a/2 : a/2\sqrt{2} : a/\sqrt{3}$$
$$= 1 : 1/\sqrt{2} : 2/\sqrt{3}$$
$$= 1 : 0.701 : 1.154$$

(iii) Body Centred Cubic lattices.

$$d_{(100)} : d_{(110)} : d_{(111)} = 1 : 2/\sqrt{2} : 1/\sqrt{3}$$
$$= 1 : 1.414 : 0.577$$