* Derive Einstein's mass energy relation and Discuss it's consequences.

The mass energy equivalence relation is expressed by,

$$E = mc^2 \quad \ldots\ldots\ldots\ldots (i)$$

where m = mass

and c = velocity of light in vacuum.

This relation is derived by the use of ordinary mechanics and special theory of relativity.

From ordinary mechanics we have Newton's definition of force expressed by rate of change of momentum,

$$F = \frac{d}{dt}(mv) \quad \ldots\ldots\ldots (ii)$$

where v is velocity of the particle. According to special theory of relativity, both mass and velocity are variable. Therfore, equation (ii) is written as,

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \ldots\ldots (iii)$$

If this force acts on the body and displaces through dx, then work done

$$dW = F \cdot dx \quad \ldots\ldots\ldots (iv)$$

Substituting value of F from equation (iii) in this, we obtain.

$$dW = m \cdot \frac{dv}{dt} \cdot dx + v \cdot \frac{dm}{dt} \cdot dx$$

or, $$dW = m \cdot dv \frac{dx}{dt} + v \cdot dm \frac{dx}{dt} \quad \ldots\ldots (v)$$

We know that, $v = \frac{dx}{dt}$ for the body. Therefore, equation (v) gives,

$$dW = mv\,dv + v^2\,dm \quad \ldots\ldots (vi)$$

The law of variation of mass with velocity is expressed as,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \ldots\ldots (vii)$$

On squaring this gives

$$m^2 = \frac{m_0^2}{\left(1 - \frac{v^2}{c^2}\right)}$$

or, $$m^2c^2 - m^2v^2 = m_0^2c^2$$

Taking differentials both sides of this and remembering that m_0 and c are constants we have,

$$c^2 \cdot 2m\,dm - v^2 \cdot 2m\,dm - m^2 \cdot 2v\,dv = 0$$

Since $m \neq 0$

Thus, $$c^2\,dm = mv\,dv + v^2\,dm \quad \ldots\ldots (viii)$$

Now R.H.S. of this and that of equation (vi) are equal therefore, we have

$$dW = c^2\,dm \quad \ldots\ldots (ix)$$

We know from S.T.R. that the rest mass m_0 of a body when accelerated to attain velocity v, becomes

moving mass m for which work-done will be W. Therefore integrating equation (ix) within limits specified above we have,

$$\int_0^W dW = c^2 \int_{m_0}^{m} dm$$

On solving we obtain,

$$W = mc^2 - m_0c^2$$

or, $$W + m_0c^2 = E = mc^2 \quad \ldots\ldots (x)$$

Finally

$$E = mc^2 \quad \ldots\ldots (xi)$$

In relation (x) the term m_0c^2 represents the internal energy for building of the body. Quantity W is the external work. Therefore, L.H.S. of equation (x) represents the total energy E which is given by equation (xi). This is known as Einstein's mass energy relation.

Importance of this energy-mass relation is building up of nucleus in which mass number is different from atomic number. It is called mass defect and expressed

$$m - A \equiv dM$$

And binding energy for the nucleus is,

$$E = c^2 \cdot dm$$

Thus this relation is of fundamental importance in building of material universe. This process is going on from the time immemorial and will be going on till the universe exists.

A devastating use of this relation was atom bomb droped on Hiroshima and Nagasaki in 1945. In that there is nuclear fission. Where Uranium brakes up in to two lighter nuclei with release of energy according to this relation ~~$E = mc^2$~~ $E = c^2 \cdot dm$ and it is still going on.

brake
break