

← TENSORS →

Elementary properties of tensors →

In mathematical

physics tensors are introduced as generalizations of Scalars (rank 0) and Vectors (rank 1) to higher dimensions, used to describe physical quantities that require multiple indices and exhibit specific transformation rules under coordinate changes.

→ Elementary properties of tensors are

1) Rank (order) →

2) Transformation properties →

3) Algebraic properties

1) Rank → (a) definition → The rank of a tensor indicates the number of indices required to describe its components.

(b) Rank 0 → Invariant under coordinate transformation ~~is~~ for example - temperature, mass, etc.

(c) Rank 1 (vector) \rightarrow Has one (2) index (A^i) and transformation like a vector.

(d) Rank 2 (Matrix) \rightarrow Has two indices (T_{ij}) and can be represented as a 2-dimensional matrix for example is stress etc.

(e) Higher Rank \rightarrow Tensors with rank 3 or higher requiring three or more indices.

<2> Transformation properties. \rightarrow

(a) Coordinate invariance - While individual tensor components (T_{ij}) change under coordinate transformations, the physical entity represented by the tensor remains the same.

(b) Transformation law \rightarrow Components transform linearly according to the Jacobian of the transformation

[b(i)] Contravariant vector (A^i) \rightarrow Transforms with upper indices.

[b(ii)] Covariant vector (A_i) \rightarrow Transforms with

lower indices. contain both (covariant and contravariant)

3. Algebraic properties →

[3(a)] → Addition / Subtraction → Tensors must be of the same rank and type [eg. (0,2) type] to be added or subtracted. The sum $C_{ij} = A_{ij} + B_{ij}$ is also a tensor.

[3(b)] - Scalar Multiplication - A tensor multiplied by a scalar (cA_{ij}) remains a tensor of the same rank.