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MJC-PHYSICS
Unit-1 | Paper-05 | sem IV

Coveriant tensar →:

: A Covariant tensar is a geometric objects with components A_i, that transfoom in the same way as besic vectors under coordinate changes, satisfying.

$A'_i = A_J \frac{\partial x^J}{\partial x'^i}$. These tensors, often called Covectors or 1-forms, repersent physical quantities like gradients $(\partial\phi/\partial x^i)$ or covariant vectors in curvilinear systems.

Some key Aspects →

(1) Difination → A Coveriant tensar has its indices in the downstairs position. It is a linear functional that take a vector as input and produces a scalar (dual space). It's physical example is (a) Gradient - the derivative of a scalar field ϕ

$$\nabla\phi = \frac{\partial\phi}{\partial x^i} \text{ is a Coverian vector.}$$

(b) Matric tensor - The metric tensor g_{ij} used to measure distances is a symetric coveriant tensor of Rank 2 (two) denoted as

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

2) Distinction from contravariant: Unlike Contravariant tensors (A^i) that transform with $\frac{\partial x'^i}{\partial x^j}$ (opposite to basis), Coveriant

tensors transform with $\frac{\partial x^j}{\partial x'^i}$ (Same as basis)

3) Index Lowering — A ~~covariant~~ contravariant tensor (V^i) can be converted into a covariant tensor (V_i) using the metric tensor

$$V_i = g_{ij} V^j$$

4) Transformation Law — If x^i are old coordinate and x'^i are new, a covariant tensor of rank 1 (vector) transform as

$$\bar{A}_i = \frac{\partial x^j}{\partial \bar{x}^i} A_j$$

For a second rank covariant tensor T_{ij}, it transform as

$$\bar{T}_{ij} = T_{KL} \frac{\partial x^K}{\partial \bar{x}^i} \cdot \frac{\partial x^l}{\partial \bar{x}^j}$$

It's common uses in physics — Covariant tensors are essential in general relativity for describing Curvature via the Riemann curvature tensor and in electromagnetism for the electromagnetic field tensor.