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MSC - Physics
Unit-1 | Paper-05 | Sem IV

Contravariant tensar → A Contravariant tensar is geometric object with components A^{μ} that transform with the upper index according to the coordinate transformation rules

$$\bar{A}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} A^{\alpha}$$

generally repersenting quantities that behave inverselly to basis vectors for example velocity acceleration etc. They are crucial in physics for ensuring equations remain invariant across different coordinate systems particularly in relativity.

It's Key Aspects →

→ Diffination and Transformation — A Set of n quantities A^{μ} ($\mu = 1, 2, \ldots n$) in a coordinate System x^{μ} transforms to $\bar{A}^{\mu}$ in System $\bar{x}^{\mu}$ via the Rule

$$\bar{A}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} A^{\alpha}$$

Where Einsteins summation convention is assumed

→ Contravariant components are identified by upper indices for example A^{μ}, $B^{\mu\nu}$, etc. and its physical example ~~ex~~ is — The 4 velocity vector in special relativity $u^{\mu} = dx^{\mu}/d\tau$ | Where τ is

Propertime is a converiant vector.

→ Rank 1 (~~Contra~~ Contravariant vector) → Represents components like position dx^μ or, Velocity $v^\mu = \frac{dx^\mu}{dt}$. If basis vectors transform coveriance ($e_\mu \rightarrow \frac{\partial x^\alpha}{\partial \bar{x}^\mu} e_\alpha$), Vectors transform converiantlly ($A^\mu \rightarrow \frac{\partial \bar{x}^\mu}{\partial x^\alpha} A^\alpha$) to keep the vector invariant.

→ Higher Rank — A contravariant tensor of rank 2(two)

$A^{\mu\nu}$, transform as

$$\bar{A}^{\mu\nu} = \frac{\partial \bar{x}^\mu}{\partial x^\alpha} \frac{\partial \bar{x}^\nu}{\partial x^\beta} A^{\alpha\beta}$$

→ Contrast with coveriant → While contravariant components change in the same way as coordinate differentials (dx^μ). Coveriant component (lower ~~d~~ indices) transforms using the inverse Jacobian $\left(\frac{\partial x^\alpha}{\partial \bar{x}^\mu}\right)$.