

## Complex Analysis:

$$ax^2 + bx + c = 0$$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = b^2 - 4ac$$

$\alpha, \beta \Rightarrow$  real & unequal ( $D > 0$ )

$\Rightarrow$  real & equal ( $D = 0$ )

$\Rightarrow$  imaginary.

$$D < 0$$

$N \rightarrow 1, 2, 3, \dots$  (Natural)

$W \rightarrow 0, 1, 2, \dots$  (whole)

$I/Z \rightarrow \dots -2, -1, 0, 1, 2, \dots$  (Integers)

$Q \rightarrow$  (Rational Nos)

$\Rightarrow \left\{ \frac{p}{q} : q \neq 0, p, q \in \mathbb{R} \right\}$

$$\frac{a}{0} = \infty$$

Terminating, Repeating

$$\mathbb{R} = \mathbb{Q}^c \cup \mathbb{Q} \quad \frac{1}{\sqrt{2}}$$

$\mathbb{Q}^c =$  non-terminating & non-repeating  
( $\pi$ )

$$x^2 + 1 = 0$$

$$x^2 = -1$$

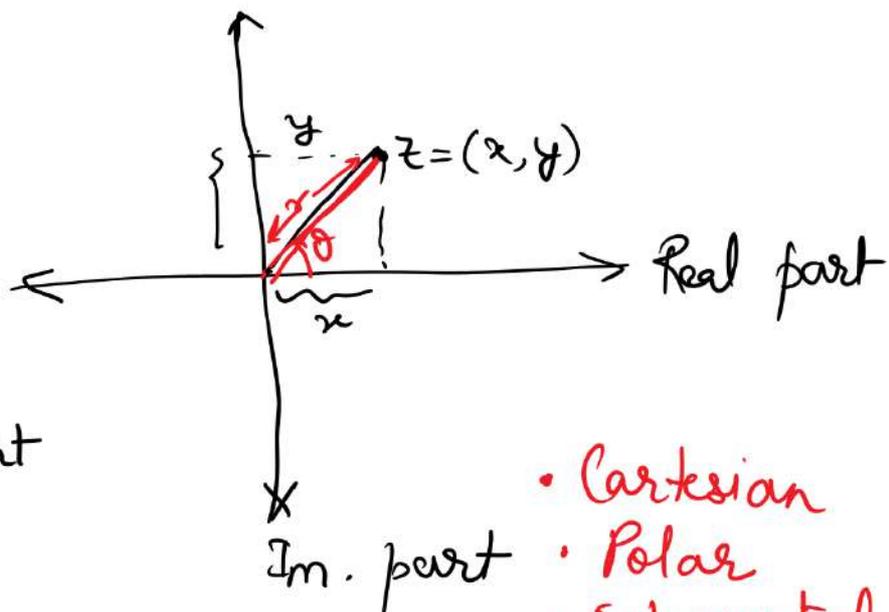
$$x = \pm\sqrt{-1}$$

$$\sqrt{-1} = i$$

$$z = x + iy \rightarrow \begin{array}{l} \text{Im. part} \\ \downarrow \\ \text{Re part} \end{array}$$

$z =$  Cartesian representation

$$z = r e^{i\theta} \quad (\text{exponential})$$



Complex no. An ordered pair of real nos:  $a, b$

$$z = (a, b) = a + ib$$

1. Equality of Comp. No.  $(a, b) = (c, d)$   
 $a + ib = c + id$  if  $a = c$  &  $b = d$

2. Sum of two Comp. No.  $z_1 = (a, b)$ ,  $z_2 = (c, d)$   
 $z_1 + z_2 = (a + ib) + (c + id) = (a + c) + i(b + d)$

3. Commutative:  $z_1 + z_2 = z_2 + z_1$   $\forall z_1, z_2 \in \mathbb{C}$  (Prove it)

$$\begin{aligned}
z_1 + z_2 &= (a+ib) + (c+id) \\
&= (a+c) + i(b+d) \\
&= (c+a) + i(d+b) \\
&= (c+id) + (a+ib) \\
&= z_2 + z_1
\end{aligned}$$

$$\because a, c, b, d \in \mathbb{R}$$

$\because$  Add<sup>n</sup> is comm. in  $\mathbb{R}$

$$\therefore a+c = c+a$$

$$\therefore b+d = d+b$$

Matrix multiplication is not comm.  $\Rightarrow AB \neq BA$

Hence complex nos. commute.

4. Addition composition is Associative.

$$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$$

### 5. Identity element. (Addition)

$$e = (0, 0)$$

$$z_1 + e = (a, b) + (0, 0) = (\underbrace{a+0}_R, \underbrace{b+0}_R) = (a, b) = z_1$$

~~$e$~~   
||

$$a + e = a$$

$$5 + 0 = 5$$
$$-2 + 0 = -2$$

$$3 \times 1 = 3$$

$$(3+i) + (i+i) = 4+2i$$

### 6. Additive inverse of Complex no.

$$z_1 + z_2 = (0, 0)$$

$$z_1 = (a, b) ; z_2 = (-a, -b)$$

↓  
Additive Inverse

- 1. closure
  - 2. ~~comm.~~
  - 3. Ass.
  - 4. ident.
  - 5. inverse
- } abelian  
gp.