Product of two complex no^s

$$z_1 \,,\, z_2 \rightarrow \text{c. no.}$$

$$z_1 = (x_1, y_1) = x_1 + iy_1$$

$$z_2 = (x_2, y_2) = x_2 + iy_2$$

$$z \cdot z_2 = (x_1 + iy_1)\cdot(x_2 + iy_2) = x_1x_2 + iy_1x_2 + ix_1y_2 - y_1y_2$$

$$= (x_1x_2 - y_1y_2) + i(y_1x_2 + x_1y_2)$$

$$z_1 \cdot z_2 = (x_1x_2 - y_1y_2 \,,\, x_1y_2 + x_2y_1)$$

M1. comm.

M2. Ass.

M3. Identity element $\rightarrow$ $(1,0)$ $\quad \alpha \cdot 1 = \alpha$

$$z_1 = x_1 + iy_1 = (x_1, y_1)$$

$$z_1 \cdot (1 + i0) = (x_1 + iy_1) \cdot (1 + i0)$$

$$= x_1 + iy_1 = (x_1, y_1)$$

M4. $\forall\, z_1 \in \mathbb{C} \quad \exists\, z_2 \in \mathbb{C}$

$$z_1 \cdot z_2 = (1,0) \Rightarrow (x_1x_2 - y_1y_2,\ x_1y_2 + x_2y_1) = (1,0)$$

$x_1x_2 - y_1y_2 = 1$; $x_1y_2 = -x_2y_1$ $\quad z_1 = (x_1, y_1)$

$x_1x_2 = 1 + y_1y_2$ $\quad \boxed{z_2 = ?}$

$\downarrow \qquad \downarrow$

$? \qquad ?$

$$x_2 = \frac{x_1}{x_1^2 + y_1^2}, \quad y_2 = \frac{-y_1}{x_1^2 + y_1^2}, \quad x_1^2 + y_1^2 \neq 0$$

$\forall\, z_1 \;\exists$ mult. inverse z_2 s.t.

$$z_2 = \left(\frac{x_1}{x_1^2 + y_1^2}, \frac{-y_1}{x_1^2 + y_1^2}\right) \quad ; \quad (x_1, y_1) \neq (0,0)$$

$\mathbb{C}$ is an abelian gp. w.r.t operation mult.

Division comp.

$$z_1 \div z_2 = z_1 (z_2)^{-1} = \frac{z_1}{z_2}$$

* Multiplication is distributive over addition.

$$z_1 \cdot (z_2 + z_3) = z_1 \cdot z_2 + z_1 \cdot z_3$$

Conjugate of a complex nos:

$z_1 = x_1 + iy_1$

$\overline{z}_1 = x_1 - iy_1$ is the conjugate of z_1

- $\overline{(z_1 \pm z_2)} = \overline{z}_1 \pm \overline{z}_2$ $\qquad (\cup)' = \cap$
- $\overline{(z_1 z_2)} = \overline{z}_1 \cdot \overline{z}_2$
- $\overline{(z_1 \div z_2)} = \dfrac{\overline{z}_1}{\overline{z}_2}$, $z_2 \neq 0$

$z = x + iy \quad , \quad \bar{z} = x - iy$

$\underset{\text{Real}}{x}$ $\underset{\text{Imag.}}{y}$

$$z + \bar{z} = 2x$$

$$x = \frac{1}{2}(z + \bar{z})$$

$$Re(z) = \frac{1}{2}(z + \bar{z})$$

$$z - \bar{z} = 2iy \Rightarrow y = \frac{1}{2i}(z - \bar{z})$$

$$Im(z) = \frac{1}{2i}(z - \bar{z})$$

$$z\bar{z} = (x+iy)(x-iy)$$

$$= x^2 + y^2$$

$$z\bar{z} = Re(z)^2 + Im(z)^2$$

* $z\bar{z} = |z|^2$

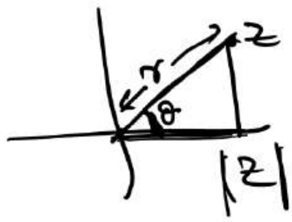
θ = Amplitude ; gen. Argument

- $\sin\theta = 0$
- $\theta = n\pi, \; n = 0, 1, 2 \ldots$

* Principal Argument : $-\pi < \theta \leq \pi$

$$z = r e^{i\theta}$$

$$z = r(\cos\theta + i\sin\theta)$$

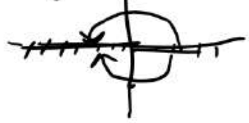


$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta)$$

$$x^2 + y^2 = r^2$$



Properties of Modulus :

$|z|^2 = z\bar{z}$

$|z_1 z_2|^2 = z_1 z_2 \cdot \overline{z_1 z_2}$

$= z_1 \bar{z}_1 \cdot z_2 \bar{z}_2$

$= |z_1|^2 |z_2|^2$

1. $|z| = 0 \iff z = 0$

2. $|z_1 z_2| = |z_1| \cdot |z_2| \quad \forall\, z_1, z_2 \in \mathbb{C}$

3. $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}, \quad \forall\, z_1 \in \mathbb{C},\ z_2 \neq 0 \in \mathbb{C}$

4. $|z_1 + z_2| \leq |z_1| + |z_2| \quad \forall\, z_1, z_2 \in \mathbb{C}$ (H.W.)