

\* What are inertial and non-inertial frames of reference?

Discuss Galilean relativity or Galilean transformation.

A frame of reference in which a static body or the body moving uniformly along a straight line is not accelerated without application of force, is called inertial frame of reference. The simplest example is; the stars including the Sun. ~~is an example of inertial frame.~~ 'Newton's third law of motion holds in such a frame.'

A frame of reference in which a static or uniformly moving body along straight line is accelerated without an applied force is called non-inertial. The Earth is a good example, where ~~Coriolis~~ <sup>Coriolis</sup> force and centrifugal force are automatically generated due to its rotation. 'In such a frame Newton's third law of motion is violated.'

Coriolis

Galilean relativity principle is valid for much small motions compared to that of light. Here time is considered as absolute. The principle states that the uniform motion of a body along a straight line with reference to an inertial frame has no effect on the mechanical processes happening in the system.

happen  
area: star

To establish the invariance of  
↓  
invariance

mechanical laws, let us consider a static reference system  $S(Oxyz)$  and moving reference system  $S'(O'x'y'z')$  with common  $x$ -axis. This is shown in figure-1.

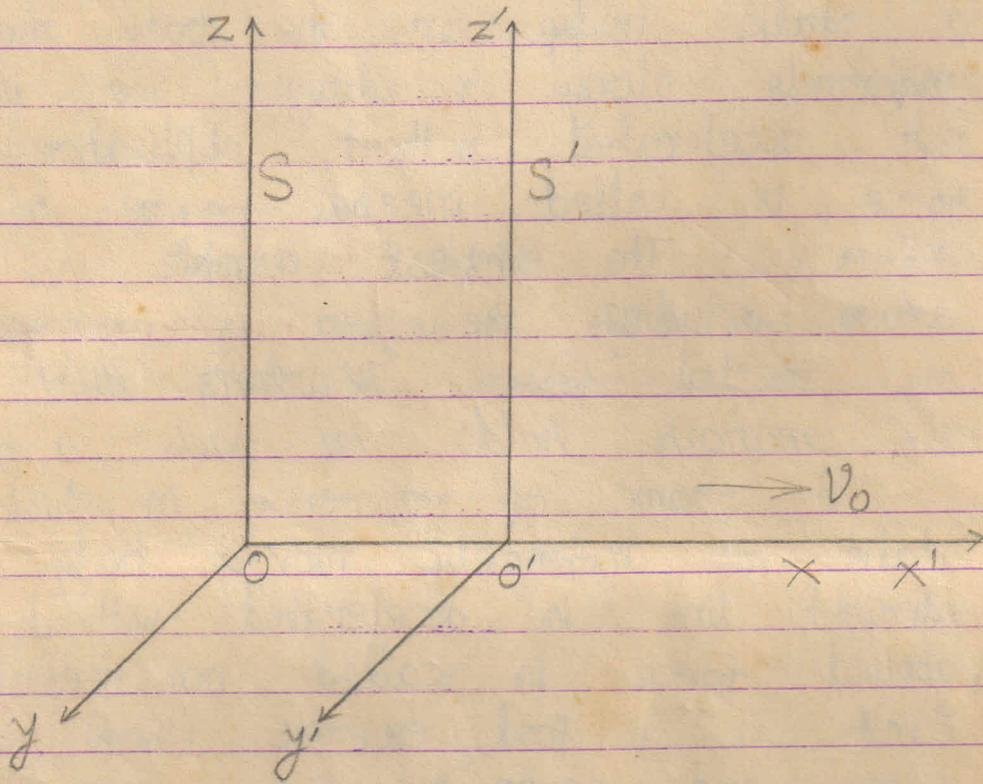


Figure  $\rightarrow$  1

Let us consider a particle  $A$  (an ant) in the moving inertial frame. For ~~convenience~~ <sup>convenience</sup> we consider two dimensional diagram with static reference frame  $S(Oxy)$  and moving reference frame  $S'(O'x'y')$ .

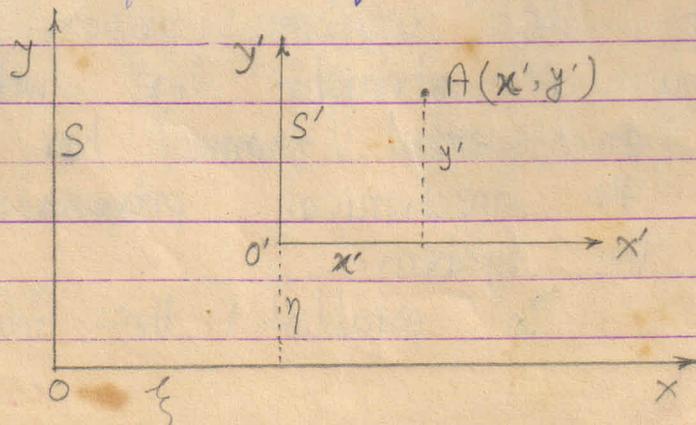


figure-2. Static  $S(Oxy)$  & Moving  $S'(O'x'y')$

Let the position of the particle in  $S'$  frame be  $A(x', y')$ . The origin of  $S'$  frame has co-ordinates  $O'(\xi, \eta)$  relative to static frame.

As indicated in figure, the  $x$  and  $y$  velocity components of frame  $S'$  relative to  $S$  is expressed by time derivatives i.e.,

$$\left. \begin{aligned} \frac{d\xi}{dt} = \dot{\xi} = \vec{U}_x \\ \text{and, } \frac{d\eta}{dt} = \dot{\eta} = \vec{U}_y \end{aligned} \right\} \dots \dots \dots (i)$$

The resultant of the two vector is,

$$\vec{U}_x + \vec{U}_y = \vec{V}_0 \dots \dots \dots (ii)$$

In  $S'$  frame the velocity of the particle (i.e. an ant) is given by,

$$\begin{aligned} V' &= \frac{dx'}{dt} + \frac{dy'}{dt} \\ \Rightarrow V' &= \dot{x}' + \dot{y}' \dots \dots \dots (iii) \end{aligned}$$

This is an observation from a point in  $S'$ .

As observe from static frame  $S$  the co-ordinates of point  $A$  is,

$$\left. \begin{aligned} x &= \xi + x' \\ y &= \eta + y' \end{aligned} \right\} \dots \dots \dots (iv)$$

The velocity components are given by their time derivatives and we obtain,

$$\left. \begin{aligned} \frac{dx}{dt} = \dot{x} = \dot{\xi} + \dot{x}' \\ \text{and } \frac{dy}{dt} = \dot{y} = \dot{\eta} + \dot{y}' \end{aligned} \right\} \dots \dots \dots (v)$$

Adding vectorial we obtain velocity of particle A relative to static frame and the result is,

$$\vec{V} = \left( \vec{\xi} + \vec{\eta} \right) + \vec{x}' + \vec{y}'$$

$$= V_0 + V'$$

This is ordinary law of relative velocity,

$$\vec{V} = \vec{V}_0 + \vec{V}' \dots \dots \dots (vi)$$

and is expressed by figure 3.

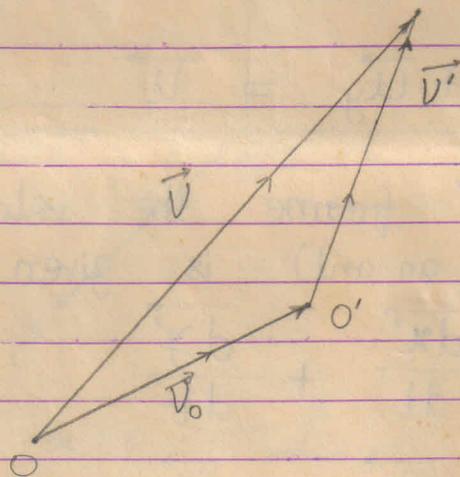


Figure - 3 representation of equation (vi)

Since velocity  $V_0$  is constant, the time derivative of quantity in equation (vi) gives

$$\frac{dV}{dt} = 0 + \frac{dV'}{dt}$$

[Since  $V_0$  is constant]

This means,

$$\vec{a} = \vec{a}' \dots \dots \dots (vii)$$

If  $m =$  mass of the particle then, we have force,

$$m\vec{a} = F = m\vec{a} \dots\dots\dots (viii)$$

Equation (vi) and (viii) establish invariance of mechanical laws under Galilean transformation.

For example, in a static train or uniformly moving train on straight track, transfer of any article from one passenger to another is not effected because, the system is inertial relative to ~~Earth~~ static Earth. While, the same train either lurches or brakes or is accelerated or negotiates the bent track the traveller experiences an extra force. The reason is non-inertiality of the moving train under such conditions.