Reduction Formulae:

$$\int \sin^n x\, dx = \int \underset{\mathrm{I}}{\sin^{n-1} x}\ \underset{\mathrm{II}}{\sin x}\, dx$$

Integrating by parts,

$$= \sin^{n-1} x \int \sin x\, dx + \int (n-1) \sin^{n-2} x \cos^2 x\, dx$$

$$= -\sin^{n-1} x \cos x + (n-1) \int \sin^{n-2} x \cos^2 x\, dx$$

$$(1 - \sin^2 x)$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x\, dx - (n-1) \int \sin^n x\, dx$$

$$(\cancel{1} + n \cancel{-1}) \int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx$$

$$\therefore \boxed{\int \sin^n x \, dx = -\frac{1}{n} \cos x \sin^{n-1} x + \frac{(n-1)}{n} \int \sin^{n-2} x \, dx}$$

Similarly,

$$\int \cos^n x \, dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\int_0^{\pi/2} \sin^n x\,dx = \overbrace{-\frac{1}{n}\cos x \sin^{n-1} x \Big|_0^{\pi/2}}^{0} + \frac{(n-1)}{n}\int_0^{\pi/2} \sin^{n-2} x\,dx$$

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{(n-1)}{n}\int_0^{\pi/2} \sin^{n-2} x\,dx \quad \text{— (1)}$$

In eq. (1) change 'n' into 'n-2'

$$\int_0^{\pi/2} \sin^{n-2} x\,dx = \frac{n-3}{n-2}\int_0^{\pi/2} \sin^{n-4} x\,dx \quad \text{—(2)}$$

putting (2) in (1)

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{(n-1)}{n} \times \frac{(n-3)}{(n-2)} \int_0^{\pi/2} \sin^{n-4} x\,dx$$

$n = 4$

$$\int_0^{\pi/2} \sin^4 x\,dx = \frac{3}{4} \times \frac{1}{2} \int_0^{\pi/2} dx$$

Similarly,

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{(n-1)}{n} \times \frac{(n-3)}{(n-2)} \times \frac{(n-5)}{(n-4)} \int_0^{\pi/2} \sin^{n-6} dx$$

Case 1: If n is even:

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n} \times \frac{(n-3)}{(n-2)} \times \frac{(n-5)}{(n-4)} \times \cdots \times \frac{1}{2} \int_0^{\pi/2} (\sin x)^0\,dx$$

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{(n-1)}{n}\frac{(n-3)}{(n-2)} \cdots \frac{1}{2} \times \frac{\pi}{2}$$

$\int \sin^3 x \, dx$

$= \frac{2}{3} \times \int_0^{\pi/2} \sin x \, dx$

$-\cos x \Big|_0^{\pi/2}$

$= 1$

Case 2: When n is odd:

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{(n-1)}{n} \times \frac{(n-3)}{(n-2)} \times \cdots \times \frac{2}{3} \int_0^{\pi/2} \sin x \, dx$$

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \times \frac{n-3}{n-2} \times \cdots \times \frac{2}{3}$$