The Complete Character table for the point group C_{3v} is given as follows.

C_{3v}	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	z	x^2+y^2, z^2
A_2	1	1	-1	R_z	
E	2	-1	0	(x, y) (R_x, R_y)	(x^2-y^2, xy) (xz, yz)
I	II			III	IV

In the upper left Corner it is the Schoenflies notation for the group and the upper row of the table Consist of the Symmetry Operations grouped in Classes.

Area I represents the Symbols for Irreducible representations according to Mulliken.

Because first two Irreducible representations are Unidimensional and hence A or B may be used.
The Character of Principal axis of rotation for both the representations are Symmetrical and hence A is used.
Subscript 1 is written for the Symmetrical character (+1) of operation σ_v, While Subscript 2 is written for Unsymmetrical Character (-1) of operation σ_v.
The Symbol E shows the two dimensional representation.

Area II :- In area II of the table are the Character of the irreducible representation of the Point groups.

Area III :- Area III gives the transformation properties of Cartesian Co-ordinates x, y, z and rotation about x, y & z axes i.e R_x, R_y and R_z.

Area IV :-
In this area IV of the table, squares and binary products of Co-ordinates according to their transformation properties are described.

The squares of the vectors (x^2+y^2) and z^2 belong to A_1, x^2-y^2 and xy taken together and xz, yz taken together belong to E.

Matrix for Identity Operation (E).

According to identity operation, the components x, y and z of a vector remain unchanged.

The equation which represents the effect of identity operation on the vector 'r' is given as-

$$E.x = 1\cdot x + 0\cdot y + 0\cdot z$$

$$Ey = 0\cdot x + 1y + 0\cdot z$$

$$E_z = 0\cdot x + 0\cdot y + 1\cdot z$$

In matrix form, these equation becomes

$$E\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Hence,

the Matrix for identity Operation E is as given below

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Character of identity Operation $= 1+1+1 = 3$

According to matrix for rotation operation the Z-co-ordinate will be unchanged by any rotation about the Z-axis. Thus, the matrix for rotation operation must be in part as,

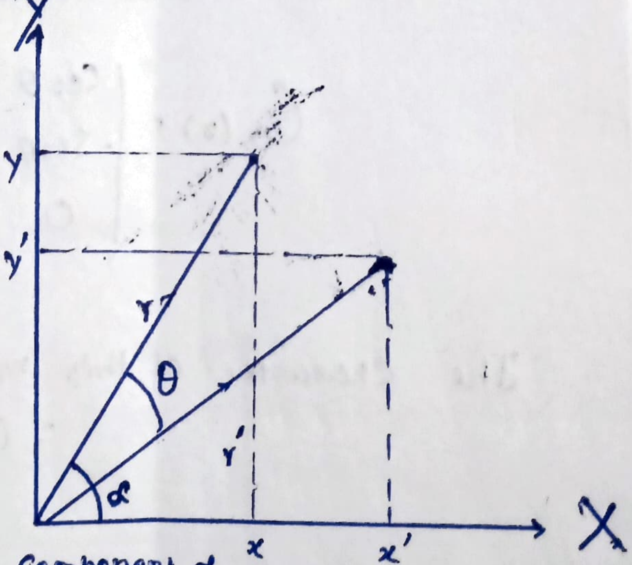
$$\begin{bmatrix} & & 0 \\ & & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, finding the four missing elements can be solved as a two dimensional problem in the xy plane.

Let us consider the vector r in the two dimensional co-ordinate system as given in the figure below.

Here the vector r can be expressed as a column matrix r.

$$r = \begin{bmatrix} x \\ y \end{bmatrix}$$



Here x and y are the component of vector r. Let the vector r is rotated clockwise through an angle θ, such that the components of the vector becomes x' and y', and the resulting vector is r'.

$$r' = \begin{bmatrix} x' \\ y' \end{bmatrix} = C_n \cdot r$$

$$\begin{aligned} x' &= r\cos(\alpha - \theta) \\ &= r(\cos\alpha \cdot \cos\theta + \sin\alpha \cdot \sin\theta) \\ &= r\cos\alpha \cdot \cos\theta + r\sin\alpha \cdot \sin\theta \end{aligned}$$

$$x' = x\cos\theta + y\sin\theta \quad \text{——— (i)}$$

and

$$\begin{aligned} y' &= r\sin(\alpha - \theta) \\ &= r\sin\alpha \cdot \cos\theta - r\cos\alpha \cdot \sin\theta \\ &= y\cos\theta - x\sin\theta \end{aligned}$$

$$y' = y\cos\theta - x\sin\theta \quad \text{——— (ii)}$$