Similarly,

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{(n-1)}{n} \times \frac{(n-3)}{(n-2)} \times \cdots \times \frac{1}{2} \times \frac{\pi}{2} \; ; \; n \text{ is even}$$

$$\int_0^{\pi/2} \cos^n x \, dx = \frac{n-1}{n} \times \frac{n-3}{n-2} \times \cdots \quad \times \frac{2}{3} \; ; \; n \text{ is odd.}$$

Q1: $\int_0^{\pi/2} \sin^7 x \, dx$

$$\frac{6}{7} \times \frac{4}{5} \times \frac{2}{3}$$

Q2: $\int_0^{\pi/2} \sin^6 x \, dx$

$$= \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \; \frac{\pi}{2}$$

$$\underbrace{\int \tan^n x \, dx}_{I_n} = \int \tan^2 x \tan^{n-2} x \, dx$$

$$= \int (\sec^2 x - 1) \tan^{n-2} x \, dx$$

$$= \int \sec^2 x \underbrace{\tan^{n-2} x \, dx}_{} - \underbrace{\int \tan^{n-2} x \, dx}_{I_{n-2}}$$

Let $\tan x = t$ so $\sec^2 x \, dx = dt$

$$\int t^{n-2} \, dt$$

$$I_n = \frac{(\tan)^{n-1} x}{n-1} - I_{n-2}$$

$$I_5 = \frac{\tan^4 x}{4} - I_3$$

$$I_3 = \frac{\tan^2 x}{2} - I_1$$

$$I_3 = \frac{\tan^2 x}{2} - \log\sec x$$

So $$\boxed{I_5 = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log\sec x}$$

$$I_1 = \int \tan x \, dx$$

$$I_1 = \int \frac{\sin x}{\cos x} dx$$

$$\cos x = t$$

$$-\sin x \, dx = dt$$

$$I_1 = -\int \frac{dt}{t} = -\ln(t)$$

$$= \log\sec x$$

$$\int \cot^n x \, dx = \frac{-1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x \, dx .$$

Q: If $I_n = \int_0^{\pi/4} \tan^n x \, dx$. Prove that

(i) $I_n + I_{n-2} = \frac{1}{n-1}$

(ii) $n\left(I_{+1} + I_{n-1}\right) = 1$