* Explain length contraction, time dilation and relativistic law of velocity addition on the basis of Lorentz - Einstein transformation.

length contraction:-

For establishing Lorentz - Fitzerald contraction, let us consider a rod whose length in stationary frame is,

$$l = x_2 - x_1 \quad \text{.....................} \quad (i)$$

The same length appears to be l' in S' such that

$$l' = x_2' - x_1' \quad \text{..........} \quad (ii)$$

We have to find the relation between l and l'.

If v = uniform velocity of S' relative to S along x-direction, then Lorentz-Einstein transformation relation gives,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{..............} \quad (iii)$$

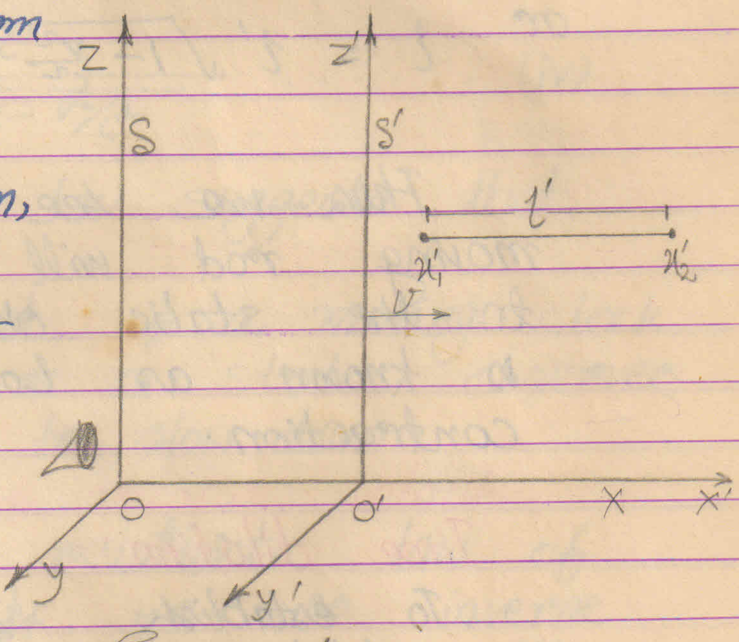


Figure-1

Using this result with x_2' and x_1' we have

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and $$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Substituting these values in equation (ii) we obtain,

$$l' = x_2' - x_1'$$
$$= \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

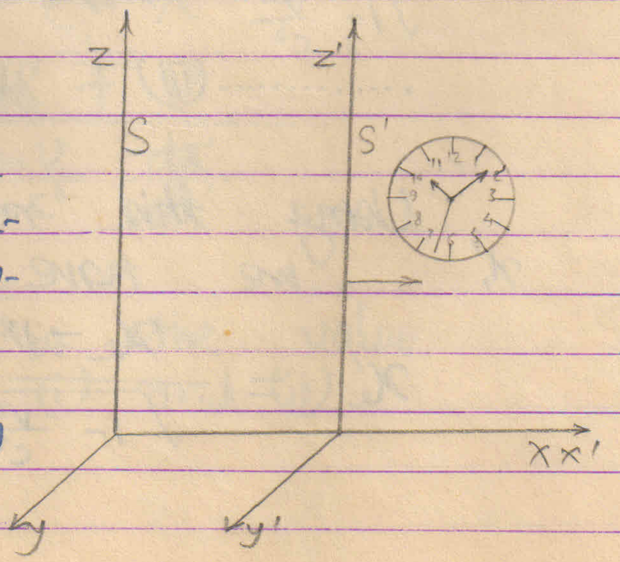
Using the value of $x_2 - x_1$ from equation (i) we obtain,

$$l' = \frac{l}{\sqrt{1 - \frac{v^2}{c^2}}}$$

or $$l = l'\sqrt{1 - \frac{v^2}{c^2}} \quad \ldots\ldots\ldots \text{(iv)}$$

Thus we see that the moving rod will appear~~s~~ shorter to the static observer. This is known as Lorentz - Fitzerald contraction.

Time dilation :-



To establish time dilation relation with the help of Lorentz-Einstein transformation we use,

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}} \quad \ldots\ldots \text{(i)}$$

where, c is velocity of light in vacuum, primed quantity is for moving frame and unprimed for static frame.

Also, for moving frame,

$$x = vt \quad \cdots\cdots\cdots \text{(ii)}$$

Using this value of x in equation (i) we obtain,

$$t' = \frac{t - v.vt/c^2}{\sqrt{1 - v^2/c^2}}$$

$$\text{or, } t' = \frac{t \times (1 - v^2/c^2)}{\sqrt{1 - v^2/c^2}}$$

$$\text{or } t' = t\sqrt{1 - v^2/c^2} \quad \cdots\cdots\cdots \text{(iii)}$$

$$\text{or } t = \frac{t'}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots \text{(iv)}$$

Equation (iii) or (iv) suggests that

$$t > t'$$

i.e., The second, of a moving clock appears dilated to a static observer. The clock will be slow down.

Relativistic law of relative velocity:

To obtain relativistic law of relative velocity, we use inverse Lorentz - Einstein transformation relations, reason is that here velocity is to be reckoned relative to the moving frame. Here we have for the static and moving frame the relation is

$$\frac{dx}{dt} = u \quad \text{for } S \quad \cdots\cdots \text{(i)}$$

$$\text{and } \frac{dx'}{dt'} = u' \quad \text{for } S' \quad \cdots\cdots \text{(ii)}$$

The inverse Lorentz-Einstein transformation relations are,

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots\cdots (iii)$$

and $$t = \frac{t' + vx'/c^2}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots (iv)$$

~~Now inverse Lorentz Einstein transformation relations are,~~

Now taking differentials of equations (iii) and (iv) both sides we obtain,

$$dx = \frac{dx' + v\,dt'}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots (v)$$

and $$dt = \frac{dt' + v\,dx'/c^2}{\sqrt{1 - v^2/c^2}} \quad \cdots\cdots (vi)$$

Now dividing equation (v) by equation (vi) we obtain,

$$\frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + v\,dx'/c^2}$$

or, $$\frac{dx}{dt} = \frac{dx'/dt' + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}}$$

Using in this the value from equation (i) and (ii) we obtain,

$$\boxed{U = \frac{U' + v}{1 + U'v/c^2}} \quad \ldots\ldots\ldots (vii)$$

This is known as relativistic relative velocity. It is valid for velocity v comparable to that of light. For practical life.

$$V << C$$

Thus $\frac{V}{C^2} \longrightarrow 0$

And we obtain.

$$\boxed{U = U' + v} \quad \ldots\ldots\ldots (viii)$$

This is ordinary law of vector addition.