

Phase Space

Phase space is the set of all possible positions (r) and momenta (p) of a physical system. It is a fundamental concept in classical statistical mechanics used to describe the microscopic state of a system. It is a multidimensional mathematical space where each axis represents a coordinate of position or momentum for the particles in a system. An instantaneous state of a single particle is represented by a single point in this space, known as phase point or representative point (coordinates $x, y, z, p_x, p_y, p_z, \dots$). 6 (six) dimensions (3 for position and 3 for momentum) are called single particle (3 dimensions) and $6N$ dimensions ($3N$ position, $3N$ momentum) are called N particles (5D). Phase space is ~~the~~ divide in two types

- 1) μ -space (Molecular phase space) - A 6 dimensional space used to describe the state of a single particle.

2 - Γ -space (Gas phase space) - A 6N dimensional space used to describe the state of the entire system of N particles.

✓ Phase Trajectory - As a system evolves in time, its phase point moves, tracing a path known as a phase trajectory.

≈ Liouville's Theorem - The density of system points in phase space remains constant in time as the system evolves (incompressible flow) which is foundational for understanding equilibrium.

≈ phase volume (Ω) - A small region in phase space is defined by its volume element

$$d\Omega = dx, dy, dz, dp_x, dp_y, dp_z$$

Examples of phase space -

~~# 1D (S.H.O)~~

1) 1D (S.H.O) -

$H = \frac{p^2}{2m} + \frac{1}{2}kx^2$ the phase trajectory is an ellipse in the (q, p) plane.

2) free particle in a box - The phase space volume available depends on the energy and volume of the container, calculated as

$$4\sqrt{2} \pi m^{3/2} \frac{VE^{3/2}}{3}$$